Optimized Cosmic Shear Statistics

Tim Eifler, AIfA University Bonn
Martin Kilbinger, IAP Paris
Peter Schneider, AIfA University Bonn

Paris, July 6th, 2007

Outline of the talk

Introduction to my work

Argelander-Institut für Astronomie

Goal

Find an improved cosmic shear data vector

- high information content → tight constraints on cosmological parameters
- robust against contamination of the signal (B-modes)
- small correlation between data points of different angular scales (covariance matrices)

Two-point correlation function

$$\boldsymbol{\xi} = (\xi_+(\vartheta_1), \ldots.., \xi_+(\vartheta_m), \xi_-(\vartheta_1), \ldots., \xi_-(\vartheta_m))$$

Aperture mass dispersion

$$\langle \mathbf{M}^2_{\mathrm{ap}} \rangle = (\langle M^2_{\mathrm{ap}} \rangle(\theta_1), \ldots. \langle M^2_{\mathrm{ap}} \rangle(\theta_n))$$

Second-order cosmic shear measures

Two-point correlation function (2PCF)

$$\xi_\pm(\vartheta) = \langle\gamma_t\gamma_t\rangle(\vartheta) \pm \langle\gamma_\times\gamma_\times\rangle(\vartheta)$$

Relation to the power spectrum $P_\kappa = P_\mathrm{E} + P_\mathrm{B}$

$$\begin{aligned}
\xi_+(\vartheta) &= \frac{1}{2\pi}\int_0^\infty \mathrm{d}\ell\, \ell\, J_0(\ell\vartheta)\,[P_\mathrm{E}(\ell) + P_\mathrm{B}(\ell)] \\
\xi_-(\vartheta) &= \frac{1}{2\pi}\int_0^\infty \mathrm{d}\ell\, \ell\, J_4(\ell\vartheta)\,[P_\mathrm{E}(\ell) - P_\mathrm{B}(\ell)]
\end{aligned}$$

Important

$\xi_\pm$ are filtered versions of the power spectrum P_κ. The filter functions are the Bessel functions J_0 and J_4

Aperture mass dispersion

Aperture mass dispersion ($\langle M_{\rm ap}^2 \rangle$)

$$\langle M_{\rm ap}^2 \rangle(\theta) = \frac{1}{2\pi} \int_0^\infty \mathrm{d}\ell \, \ell \, P_{\rm E}(\ell) W_{\rm ap}(\theta\ell)$$

$$\langle M_{\perp}^2 \rangle(\theta) = \frac{1}{2\pi} \int_0^\infty \mathrm{d}\ell \, \ell \, P_{\rm B}(\ell) W_{\rm ap}(\theta\ell)$$

Important

1. $\langle M_{\rm ap}^2 \rangle$ is also a filtered version of the power spectrum P_κ
2. $\langle M_{\rm ap}^2 \rangle$ can be calculated from the 2pcf $\longrightarrow$ 2pcf can be seen as the basic quantity

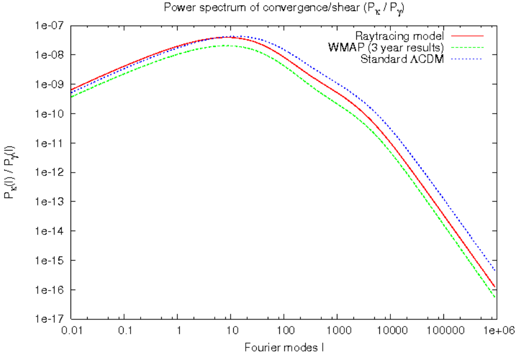
$$\langle M_{\rm ap}^2 \rangle(\theta) = \int_0^{2\theta} \frac{\mathrm{d}\vartheta \, \vartheta}{\theta^2} \, \frac{1}{2} \left[\xi_+(\vartheta) T_+ \left(\frac{\vartheta}{\theta} \right) + \xi_-(\vartheta) T_- \left(\frac{\vartheta}{\theta} \right) \right]$$

(Dis)Advantages of the different measures

Argelander-Institut für Astronomie

$\xi_\pm$

- very broad filter $\longrightarrow$ ξ_+ probes a wide range of P_κ
- includes information on angular scales larger than the size of the survey



$\langle M_{ap}^2 \rangle$

- narrowest filter function
- gives highly localized measure of P_κ
- low correlation between data points of different scales
- no large scale information of the power spectrum due to its narrow filter
- difficult to measure from data field due to gaps, holes, stars

The combined data vector

Argelander-Institut für Astronomie

Combined data vector

idea: add one value of $\xi_+(\theta_0)$ to a $\langle M_{\rm ap}^2 \rangle$ data vector to include the large scale information of P_κ

$$\mathcal{N} = (\langle M_{\rm ap}^2 \rangle(\theta_1), ...\langle M_{\rm ap}^2 \rangle(\theta_n), \xi_+(\theta_0))$$

Two-point correlation function

$$\boldsymbol{\xi} = (\xi_+(\vartheta_1), ..., \xi_+(\vartheta_m), \xi_-(\vartheta_1), ..., \xi_-(\vartheta_m))$$

Aperture mass dispersion

$$\langle \mathbf{M}_{\rm ap}^2 \rangle = (\langle M_{\rm ap}^2 \rangle(\theta_1), ...\langle M_{\rm ap}^2 \rangle(\theta_n))$$

Comparing the information content

Argelander-Institut für Astronomie

Method

Information content of a cosmic shear data vector → ability of constraining cosmological parameters → Bayesian likelihood analysis

Bayes theorem - cosmic shear data

$$P(\boldsymbol{\pi}|\boldsymbol{\xi}_{\pm},\Lambda CDM) = \frac{P(\boldsymbol{\xi}_{\pm}|\boldsymbol{\pi},\Lambda CDM)\, P(\boldsymbol{\pi}|\Lambda CDM)}{P(\boldsymbol{\xi}_{\pm}|\Lambda CDM)}$$

Likelihood

$$P(\boldsymbol{\xi}|\boldsymbol{\pi},\Lambda CDM) = \frac{1}{(2\pi)^{n/2}\sqrt{\det \mathbf{C}_{\xi}}}\, exp\left[-\frac{1}{2}(\boldsymbol{\xi}(\boldsymbol{\pi}) - \boldsymbol{\xi}^{f})^{t}\ \mathbf{C}_{\xi}^{-1}\ (\boldsymbol{\xi}(\boldsymbol{\pi}) - \boldsymbol{\xi}^{f})\right]$$

Marginalization

$$P(\boldsymbol{\pi}_{12}|\boldsymbol{\xi}_{\pm},\Lambda CDM) = \int d\pi_3 \int d\pi_4\, P(\boldsymbol{\pi}_{1234}|\boldsymbol{\xi}_{\pm},\Lambda CDM)$$

Inversion of the covariance

First guess

$$\hat{\mathbf{C}}_*^{-1} = \left(\hat{\mathbf{C}}^{ML}\right)^{-1}$$

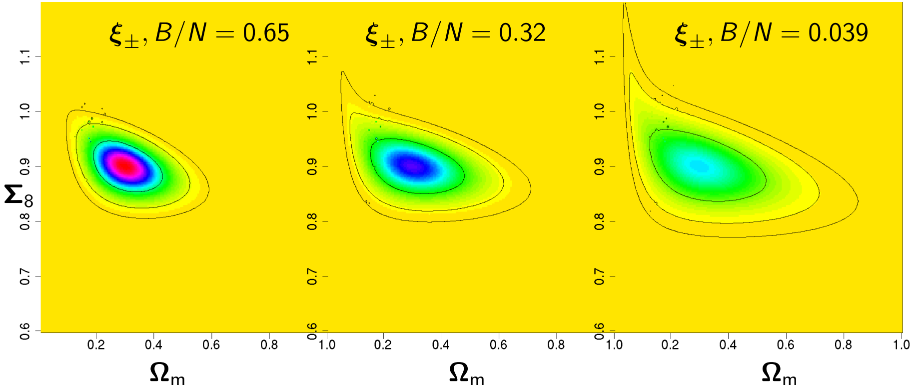
- estimator is consistent but it is *biased* due to noise in $\hat{\mathbf{C}}$.
- only linear transformations preserve "unbiasedness"

The amount of bias/size of the likelihood contours vary dependent on the relation

$$\frac{\text{number of bins (B)}}{\text{number of independent realizations (N)}}$$

- more realisations $\longrightarrow$ larger contours
- more bins $\longrightarrow$ smaller contours
- for $B \geq N - 2$ the covariance matrix becomes singular

Without correction factor...one example



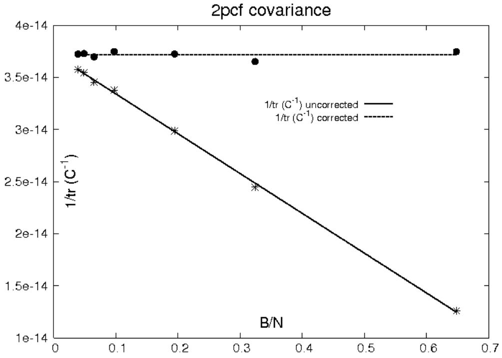
Correction-Factor (Hartlap et al. 2006; Anderson 2003)

An unbiased estimator for the inverted covariance is given by

$$\hat{\mathbf{C}}^{-1} = \frac{N - B - 2}{N - 1} \hat{\mathbf{C}}_*^{-1} \quad \text{for } B < N - 2$$

Checking the correction factor

Argelander-Institut für Astronomie

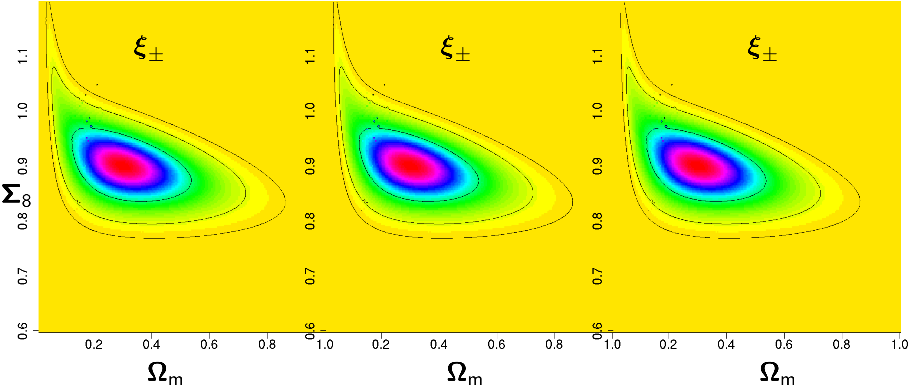


- ray-tracing simulations provide 36 independent realisations
- multiply the number of independent realisations by adding different Gaussian noise to the galaxy ellipticities ($N = 108, 216,$ $360, 720, 1080,$ $1440, 1800$)

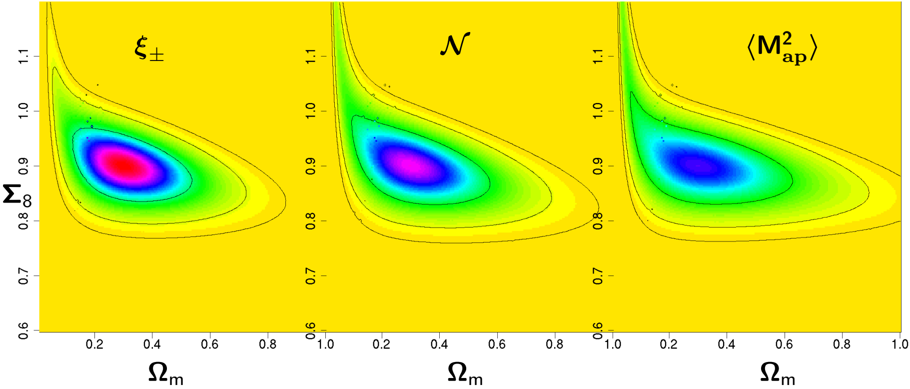
- estimate the covariance from every sample
- consider the trace of the inverted covariance depending on the ratio bins/realisations

Now it works...

1. correction factor = 0.34
2. correction factor = 0.67
3. correction factor = 0.96

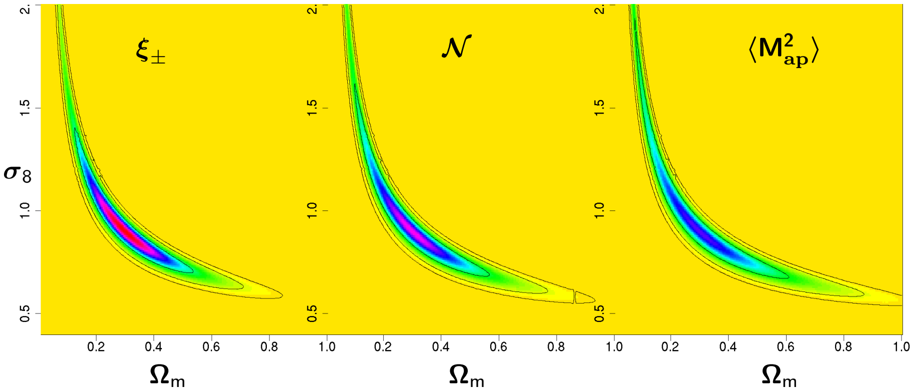


Solution and results



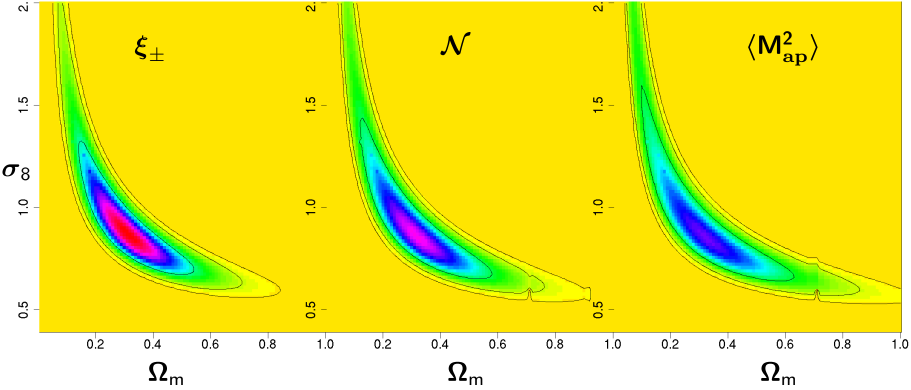
- 2pcf: ϑ-range$=0.2' - 199'$, 35 bins each for $\xi_\pm$
- $\mathcal{N}$: added data point $\xi_+(5')$, 21 bins
- $\langle M_{ap}^2 \rangle$: θ-range$=2.2' - 99'$, 20 bins

More results



parameter space	$\langle M_{ap}^2 \rangle$	$\mathcal{N}$	ξ	$\Delta\mathcal{N}$	$\Delta\xi$
Γ vs. Ω_m	14.7	11.7	9.1	20.4 %	38.1 %
σ_8 vs. Γ	23.1	19.0	14.6	17.8 %	36.8 %
σ_8 vs. Ω_m	427.1	314.5	220.1	26.4 %	48.5 %
z_0 vs. Ω_m	46.4	41.0	32.9	11.6 %	29.1 %

Marginalized over z_0



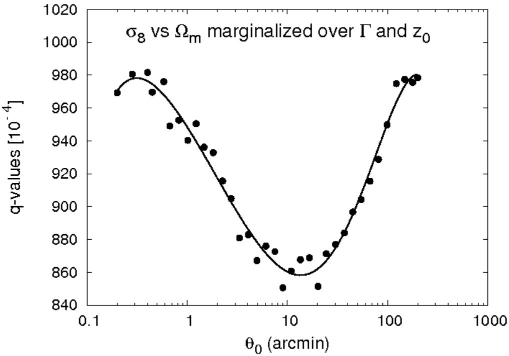
parameter space	$\langle M_{ap}^2 \rangle$	$\mathcal{N}$	ξ	$\Delta\mathcal{N}$	$\Delta\xi$
σ_8 vs. Ω_m (z_0)	416.9	313.4	230.0	25.8 %	44.8 %
σ_8 vs. Ω_m (Γ)	780.5	720.9	527.0	7.6 %	32.5 %
σ_8 vs. Ω_m (Γ, z_0)	983.8	850.6	623.5	13.5 %	36.6 %

Optimizing the combined data vector $\mathcal{N}$

Argelander-Institut für Astronomie

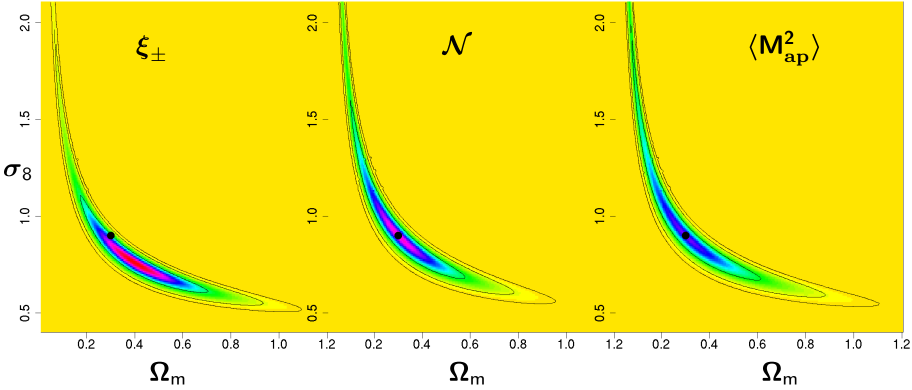
$$\mathcal{Q}_{ij} \equiv \frac{\int \mathrm{d}^2\pi \; P(\pi_1,\pi_2)(\pi_i - \pi_i^{\mathrm{f}})(\pi_j - \pi_j^{\mathrm{f}})}{\int \mathrm{d}^2\pi \; P(\pi_1,\pi_2)}$$

$$q = \sqrt{\det \mathcal{Q}_{ij}} = \sqrt{\mathcal{Q}_{11}\mathcal{Q}_{22} - \mathcal{Q}_{12}^2}$$



Idea: Vary the added $\xi_+(\theta_0)$ in $\mathcal{N}$ in order to optimize the information content

Contamination with B-modes



- The likelihood maximum of the 2PCF data vector is far away from the "true" cosmological parameters
- The combined data vector is hardly affected by the contamination
- It still gives better constraints on cosmological parameters than the aperture mass dispersion

Conclusions and Outlook

The combined data vector

- The new data vector $\mathcal{N}$ is a strong improvement in information content compared to $\langle M_{\mathrm{ap}}^2 \rangle$
- $\mathcal{N}$ can be optimized by varying $\xi_+(\theta_0)$
- its covariance matrix is much more diagonal compared with the 2PCF $\rightarrow$ more robust against numerical problems during the inversion
- it is hardly affected by a B-mode contamination on small angular scales

Future work

- Improve information content further by looking into other cosmic shear statistics (e.g. ring statistics)
- higher order statistics
- Find better estimates for covariances (robust, unbiased)

Ingredients for the likelihood analysis

Data vectors and covariances

- calculate power spectrum P_δ for our fiducial model according to Smith et al.2003
- calculate P_κ and the data vectors $\xi^f, \langle \mathbf{M}^2_{\rm ap} \rangle^f, \mathcal{N}^f$
- vary parameters and recalculate the data vectors $\xi(\pi), \langle \mathbf{M}^2_{\rm ap} \rangle(\pi), \mathcal{N}(\pi)$ for every variation
- derive the covariance matrix $\mathbf{C}_\xi$ by field to field variation from ray-tracing simulations (Jenkins et al. 2001 for simulation details and Hamana & Mellier 2001 for details of the ray-tracing algorithm)
- calculate therefrom the covariances of $\langle \mathbf{M}^2_{\rm ap} \rangle$ and $\mathcal{N}$

Fiducial model

$\Omega_{\rm m}$	Ω_Λ	h	Γ	σ_8	$\Omega_{\rm b}$
0.3	0.7	0.7	0.1723	0.9	0.04

Covariances of ξ: including B-modes

Argelander-Institut für Astronomie

Recall

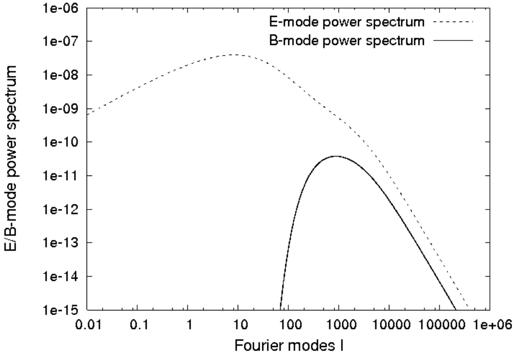
$$\langle \Delta P_E(\bar{\ell}) \Delta P_B(\bar{\ell}') \rangle = 0 \;\rightarrow\; \mathbf{C}_{tot} = \mathbf{C}_E + \mathbf{C}_B$$

$$\begin{aligned}
\langle \Delta\xi_\pm(\theta_1)\; \Delta\xi_\pm(\theta_2) \rangle &= \int_0^\infty \frac{\mathrm{d}\ell\; \ell}{\pi A}\; J_{0/4}(\ell\theta_1) J_{0/4}(\ell\theta_2) \\
&\times \left\{ \left(P_E(\ell) + \frac{\sigma_\epsilon^2}{2n} \right)^2 + \left(P_B(\ell) + \frac{\sigma_\epsilon^2}{2n} \right)^2 \right\} \\
\langle \Delta\xi_+(\theta_1)\; \Delta\xi_-(\theta_2) \rangle &= \int_0^\infty \frac{\mathrm{d}\ell\; \ell}{\pi A}\; J_0(\ell\theta_1) J_4(\ell\theta_2) \\
&\times \left\{ \left(P_E(\ell) + \frac{\sigma_\epsilon^2}{2n} \right)^2 - \left(P_B(\ell) + \frac{\sigma_\epsilon^2}{2n} \right)^2 \right\}
\end{aligned}$$

A model for P_{B}

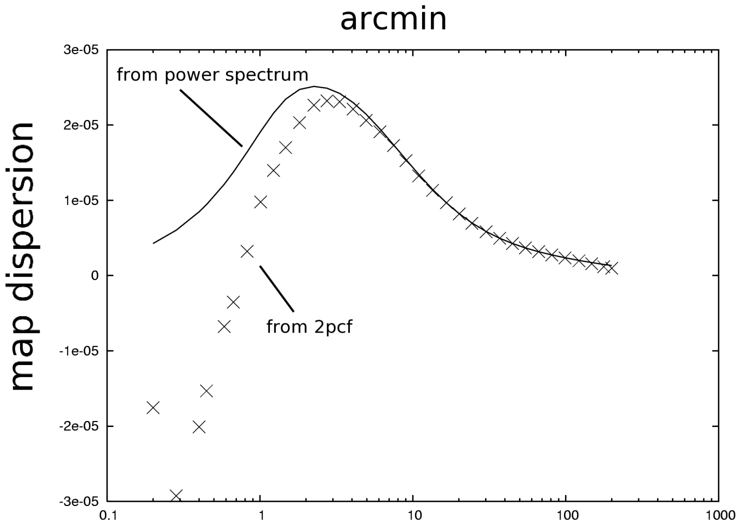
Argelander-Institut für Astronomie

- currently there is no model available for B-modes
- we know, B-modes occur mainly on small scales
- Our model:
  $P_{\mathrm{B}} = 0.2\, P_{\mathrm{E}} \exp\left(\frac{-\ell_B}{\ell}\right)$



Difficulties I

For small θ-values $\mathrm{Cov}_{\mathcal{M}}$ cannot be calculated properly from the Cov_{ξ} $\longrightarrow$ data vectors $\mathcal{M}$ and $\mathcal{N}$ start at a given $\theta_{\min}$



Problems with covariances again

Argelander-Institut für Astronomie

- Recalculate all data vectors taking P_B into account
- Calculate the covariance matrix for the correlation function where
  1. $\mathbf{C}_E$ is taken from the ray-tracing simulations
  2. $\mathbf{C}_B$ is calcualted directly from P_B
- Carefull: $\mathbf{C}_E$ needs correction factor for the inversion, $\mathbf{C}_B$ does not $\rightarrow$

$$\begin{aligned}\mathbf{C}_{\text{tot}}^{-1} &= \left(\mathbf{C}_E + \mathbf{C}_B\right)^{-1} \\ &= \mathbf{C}_E^{-1} - \frac{1}{\text{trace}\left(\mathbf{C}_E^{-1}\mathbf{C}_B\right)}\mathbf{C}_B\mathbf{C}_E^{-1}\mathbf{C}_B\end{aligned}$$

- calculate covariances for the two other data vectors from $\mathbf{C}_\xi$
- calculate the posterior likelihood and plot contours