Limits on the detectability of the CMB B–mode polarization imposed by foregrounds

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Detection of primordial gravitational waves by CMB

In inflationary scenario, the amplitude of temperature anisotropies produced by tensor perturbations (gravitational waves) is directly related to energy of inflation

$$\frac{E_i^4}{m_{Pl}^4} = 1.65 f_{(0)T}^{-1}(\Omega_\Lambda)\, Q_T^2$$

and in terms of $r = Q_T^2/Q_S^2$ $(Q_S \simeq 18\mu K)$

$$E_i \simeq 3 \times 10^{16}\, r^{1/4}\, \mathrm{GeV}$$

- Unless $r > 0.1$, temperature anisotropies can provide only upper limits on r (from WMAP $r < 0.71$)
- CMB B–mode polarization is generated only by tensor perturbations

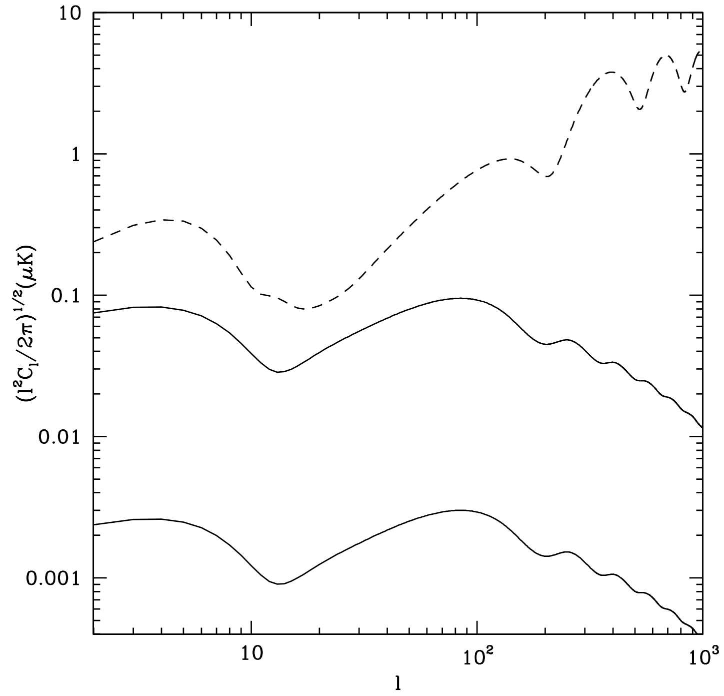
The CMB B–mode power spectrum

• $C_{B\ell}$ peaks at the angular scale corresponding to the horizon at recombination, $\ell_{peak} \simeq 90$

• The amplitude is related to the energy scale of inflation

$$\Delta B_{peak} \equiv \left[\frac{\ell(\ell+1)}{2\pi} C_{B\ell}\right]^{1/2}_{\ell=\ell_{peak}} \sim$$

$$\sim 0.3\ r^{1/2}\ \mu\mathrm{K} \simeq 0.03 \left(\frac{E_i}{10^{16}\mathrm{GeV}}\right)^2 \mu\mathrm{K}$$



• $C_{B\ell}$ peaks at $\ell < 20$ due to reionization

Problems for detecting polarization induced by gravitational waves

$\Longrightarrow$ Cosmological B–mode polarization is very weak (in very optimistic cases, $B_{\rm rms} \sim 0.1\,\mu$K)

$\Longrightarrow$ Effects mixing CMB E– and B–modes

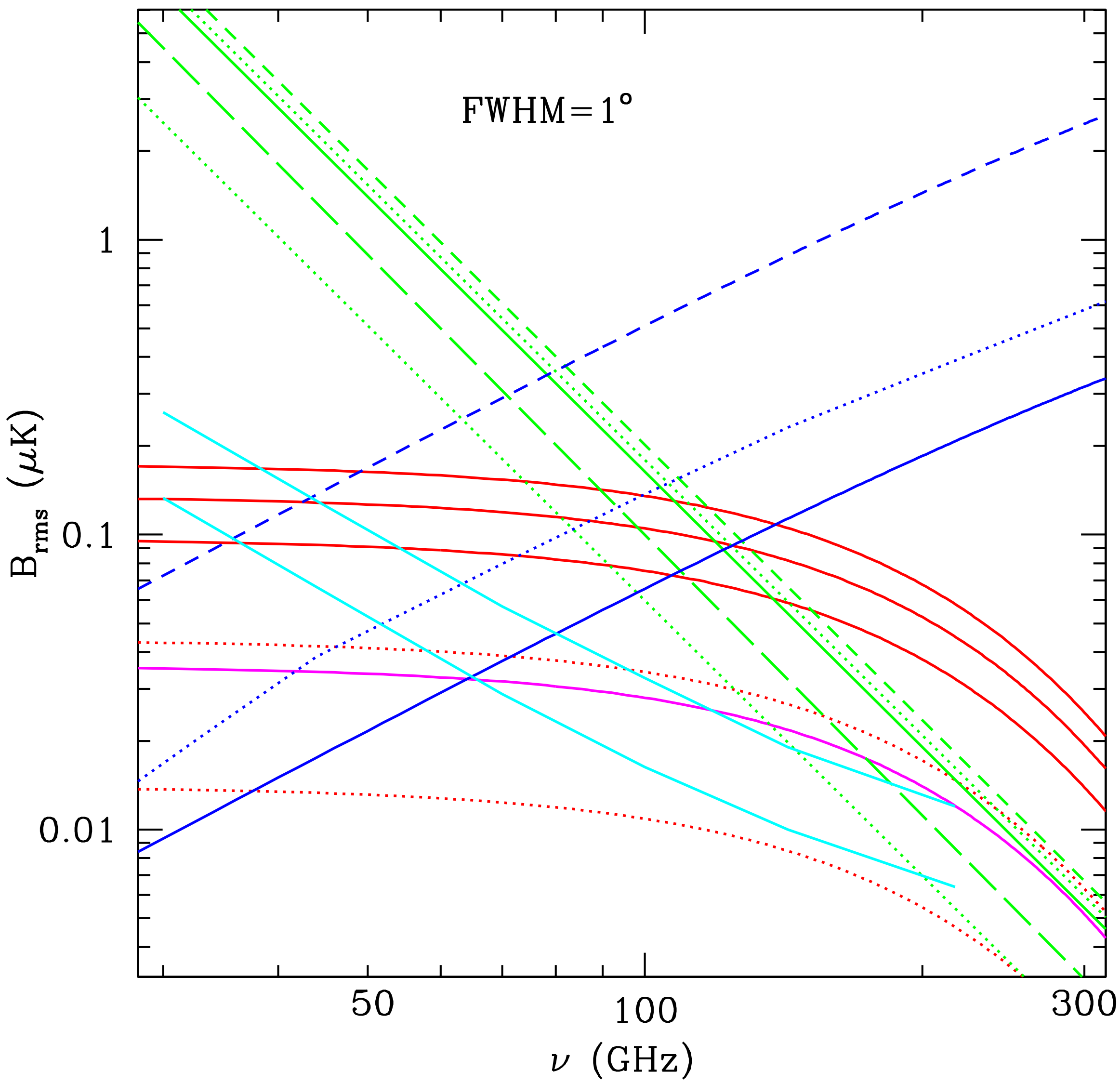
$\Longrightarrow$ Foregrounds contamination:

- high degree of polarization compared to CMB
- no difference between E– and B–mode

Galactic foregrounds: Synchrotron and Dust emission

Extragalactic foregrounds: Radio sources and lensing–induced polarization

CMB and foregrounds $B_{\rm rms}$



- **CMB**: $\tau = 0.01$, 0.1, 0.17 and $r = 0.1$ (solid lines);
  $\tau = 0.1$ and $r = 0.01$, 0.001 (dotted lines).

- **Synchrotron**: 10%–30% of the **WMAP** $\Delta T/T_{\rm rms}$ (dotted lines);
  1.4–GHz polarization data (**Brouwn&Spoelstra 1976**) (dashed line);
  from high–resolution low–latitude polarization surveys (1.4–2.7GHz;
  Duncan et al. 1997, 1999, Uyaniker et al. 1999) (solid line);
  from observations of high–latitude polarization (**Effelsberg Telescope 1.4GHz; Abidin et al. 2003**) (long–dashed line).

- **Dust**: 5% of the **WMAP** $\Delta T/T_{\rm rms}$ (dashed line);
  5% of the "100μm–map" $\Delta T/T_{\rm rms}$ (**Finkbeiner et al. 1999**) (dotted line);
  from the dust polarized emission model by **Prunet et al. (1998)** (solid line).

- **Radio sources**: estimates of **Tucci et al. (2004)**, removing all the sources with $S > 1$ or 0.2 Jy.

Galactic foregrounds subtraction

- Free–noise data at the "observational" frequency ν_o
- A foreground template is available at frequency ν_t (free–noise)
- The frequency spectrum can be approximated by a power law:

$$I_\nu(\hat{\mathbf{n}}) = I_{\nu_t}(\hat{\mathbf{n}}) \left(\frac{\nu}{\nu_t} \right)^{-\beta(\hat{\mathbf{n}})}$$

- $\beta(\hat{\mathbf{n}})$ known with uncertainty $\Delta\beta(\hat{\mathbf{n}})$

$\Longrightarrow$ Galactic foreground removed by a linear subtraction

Residual Galactic foreground at the "observational" frequency ν_o:

$$\begin{aligned}
\Delta I_{\nu_o}(\hat{\mathbf{n}}) &= I_{\nu_o}(\hat{\mathbf{n}}) - \tilde{I}_{\nu_o}(\hat{\mathbf{n}}) = \\
&= I_{\nu_t}(\hat{\mathbf{n}})\Big(\nu_o/\nu_t\Big)^{-(\beta(\hat{\mathbf{n}})+\Delta\beta(\hat{\mathbf{n}}))} - I_{\nu_t}(\hat{\mathbf{n}})\Big(\nu_o/\nu_t\Big)^{-\beta(\hat{\mathbf{n}})} \\
&= \tilde{I}_{\nu_o}(\hat{\mathbf{n}})\Big[\Big(\nu_o/\nu_t\Big)^{-\Delta\beta(\hat{\mathbf{n}})} - 1\Big] \simeq \\
&\simeq \ln\Big(\nu_t/\nu_o\Big)\tilde{I}_{\nu_o}(\hat{\mathbf{n}})\Delta\beta(\hat{\mathbf{n}})
\end{aligned}$$

Residual polarization $\Delta Q(\hat{\mathbf{n}}) \simeq \ln(\nu_t/\nu_o)\tilde{Q}(\hat{\mathbf{n}})\Delta\beta(\hat{\mathbf{n}})$

$$\Delta U(\hat{\mathbf{n}}) \simeq \ln(\nu_t/\nu_o)\tilde{U}(\hat{\mathbf{n}})\Delta\beta(\hat{\mathbf{n}})$$

B–mode power spectrum of residual Galactic foreground

- $(Q \pm iU)(\hat{\mathbf{n}}) = \sum_{\ell m} a_{\pm 2,\ell m} \; {}_{\pm 2}Y_{\ell m}(\hat{\mathbf{n}})$

  $\Delta\beta(\hat{\mathbf{n}}) = \sum_{\ell m} a_{\beta,\ell m} Y_{\ell m}(\hat{\mathbf{n}})$
- $C_{B\ell} =< |a_{B,\ell m}|^2 >$, where $a_{B,\ell m} = i(a_{2,\ell m} - a_{-2,\ell m})/2$
- Indicating $A \equiv \ln(\nu_t/\nu_o)$, we have that

$$
\begin{aligned}
< |a^{\mathcal{R}}_{2,\ell m}|^2 > \;\; &= \;\; A^2 < \int d\Omega \, {}_2Y^*_{\ell m}(\hat{\mathbf{n}})(Q + iU)(\hat{\mathbf{n}})\Delta\beta(\hat{\mathbf{n}}) \times \\
&\quad \times \int d\Omega' \, {}_{-2}Y^*_{\ell -m}(\hat{\mathbf{n}}')(Q - iU)(\hat{\mathbf{n}}')\Delta\beta(\hat{\mathbf{n}}') >=
\end{aligned}
$$

$$
= \ldots = \frac{A^2}{4\pi} \sum_{\ell_1} (2\ell_1 + 1) < |a_{2,\ell_1 m_1}|^2 > \sum_{\ell_2} (2\ell_2 + 1) C^{\beta}_{\ell_2} \begin{pmatrix} \ell & \ell_1 & \ell_2 \\ 2 & -2 & 0 \end{pmatrix} \begin{pmatrix} \ell & \ell_1 & \ell_2 \\ 2 & -2 & 0 \end{pmatrix}
$$

$$
|\ell_1 - \ell_2| \le \ell \le \ell_1 + \ell_2
$$

$$C^{\mathcal{R}}_{B\ell} = \frac{A^2}{16\pi} \sum_{\ell_1} (2\ell_1 + 1) C_{B\ell_1} \sum_{\ell_2} (2\ell_2 + 1) C^{\beta}_{\ell_2} \begin{pmatrix} \ell & \ell_1 & \ell_2 \\ 2 & -2 & 0 \end{pmatrix} \begin{pmatrix} \ell & \ell_1 & \ell_2 \\ 2 & -2 & 0 \end{pmatrix}$$

$C_{B\ell}$: Galactic foreground spectrum computed from the template and scaled to the observational frequency

C^{β}_{ℓ} : power spectrum of spectral index error $\Delta\beta$:

(1) extrapolation by $\beta(\hat{\mathbf{n}}) = <\beta>$ $\Longrightarrow$ $\Delta\beta$ map shows structures like total–intensity template

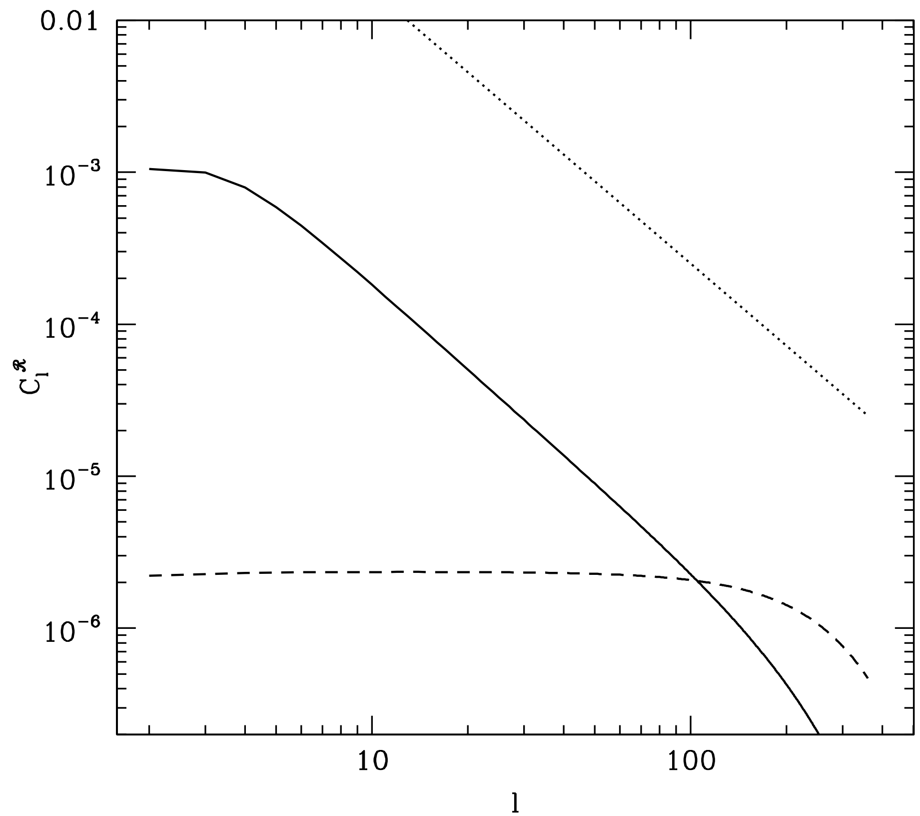
$$C^{\beta}_{\ell} \propto C^{\text{foreground}}_{I\ell} \propto \ell^{-3}$$

(2) $\Delta\beta(\hat{\mathbf{n}})$ like a white noise

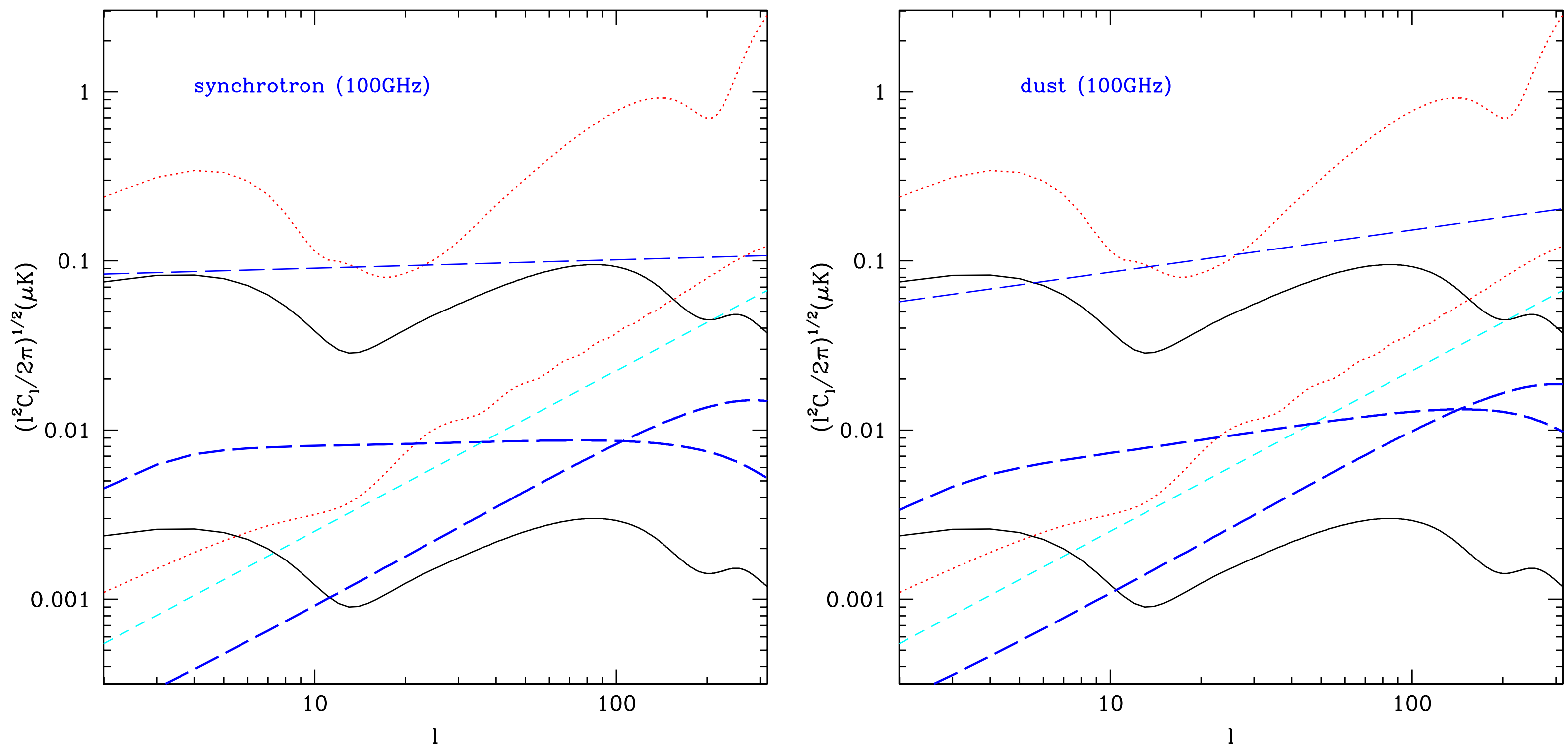
$$C^{\beta}_{\ell} = \text{constant}$$

C^{β}_{ℓ} normalization $\Longrightarrow$ $\sigma^2_{\Delta\beta} = \sum_{\ell} (2\ell + 1) C^{\beta}_{\ell} W_{\ell} / 4\pi = (0.2)^2$

Power spectrum of residual Galactic foreground



——— $C_\ell^\beta \propto \ell^{-3}$ - - - $C_\ell^\beta = \text{constant}$



Synchrotron $\quad C_{B\ell} = 1.22 \times 10^{-2}\,\ell^{-1.8}$

Dust $\quad C_{B\ell} = 4.43 \times 10^{-3}\,\ell^{-1.4}$

Radio Sources $\quad C_{B\ell} = 1.84 \times 10^{-7}\ (S_c = 200\,\mathrm{mJy})$

Uncertainty on r in a free–noise experiment

Fisher matrix estimates the minimum possible variance with which a parameter can be measured.

For a CMB polarization experiment, Fisher matrix is given by:

$$\mathcal{F}_{ij} = \sum_{\ell} \left(\frac{1}{\Delta C_{E\ell}^2} \frac{\partial C_{E\ell}}{\partial \alpha_i} \frac{\partial C_{E\ell}}{\partial \alpha_j} + \frac{1}{\Delta C_{B\ell}^2} \frac{\partial C_{B\ell}}{\partial \alpha_i} \frac{\partial C_{B\ell}}{\partial \alpha_j} \right)$$

where $\Delta C_{X\ell}^2 = \frac{2}{(2\ell+1) f_{sky}} (C_{X\ell} + N_{X\ell})^2 \qquad (X = E,\, B)$

$N_{X\ell} \Longrightarrow C_{X\ell}^{\mathcal{R}}$+ extragalactic foregrounds

Minimum possible variance $\delta\alpha_i = [\mathcal{F}_{ii}^{-1}]^{1/2}$

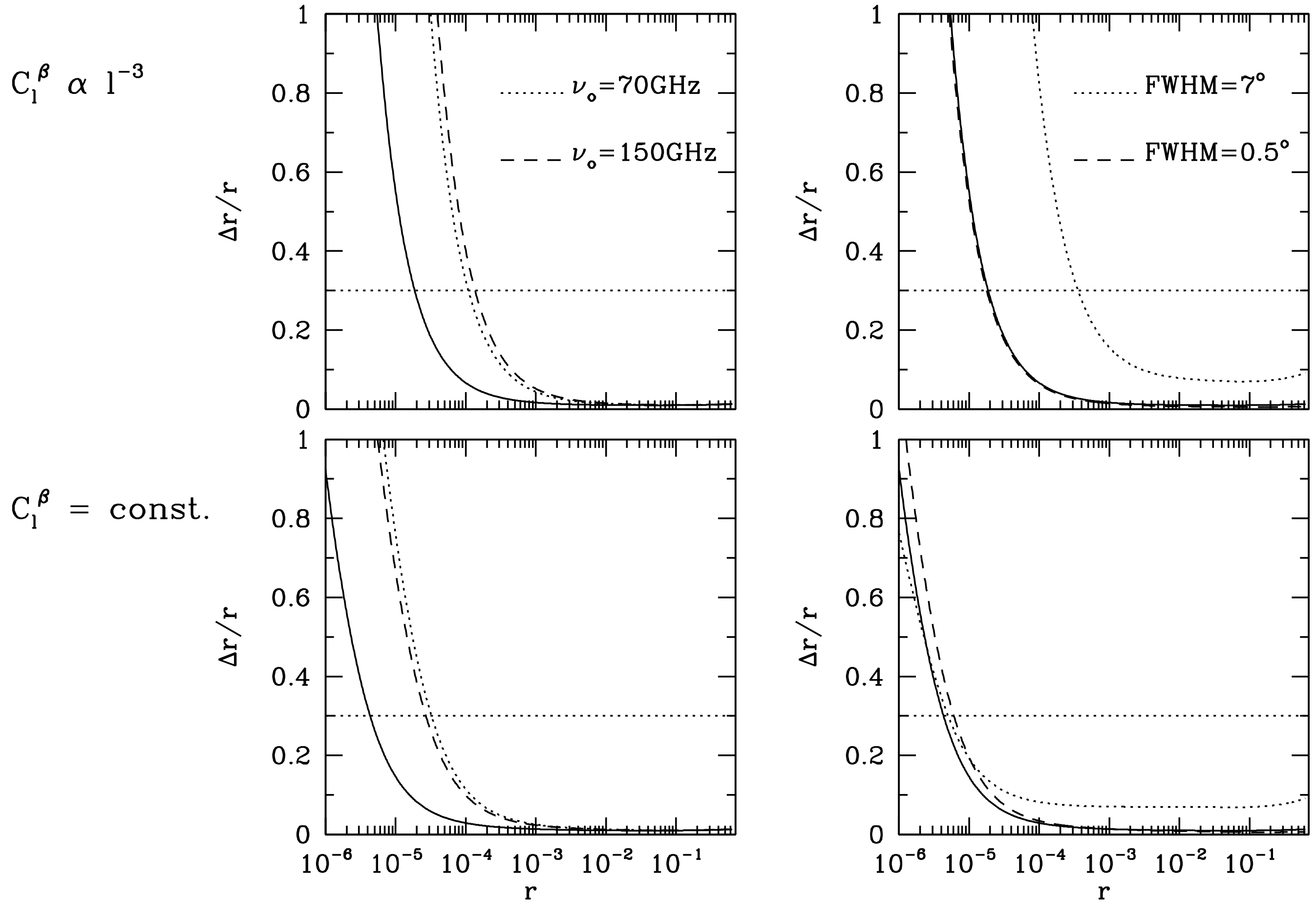
- parameters: $\alpha_{i=1,2} = \{\tau, r\}$

  (results for optical depth $\tau = 0.1$)

- Reference experiment:
  - full–sky (with Galactic–Plane cut)
  - observational frequency $\nu_o = 100\,\mathrm{GHz}$
  - template frequency: $\nu_o = 70\,\mathrm{GHz}$ (synchrotron), $\nu_o = 150\,\mathrm{GHz}$ (dust)
  - resolution FWHM$= 1^{\mathrm{o}}$

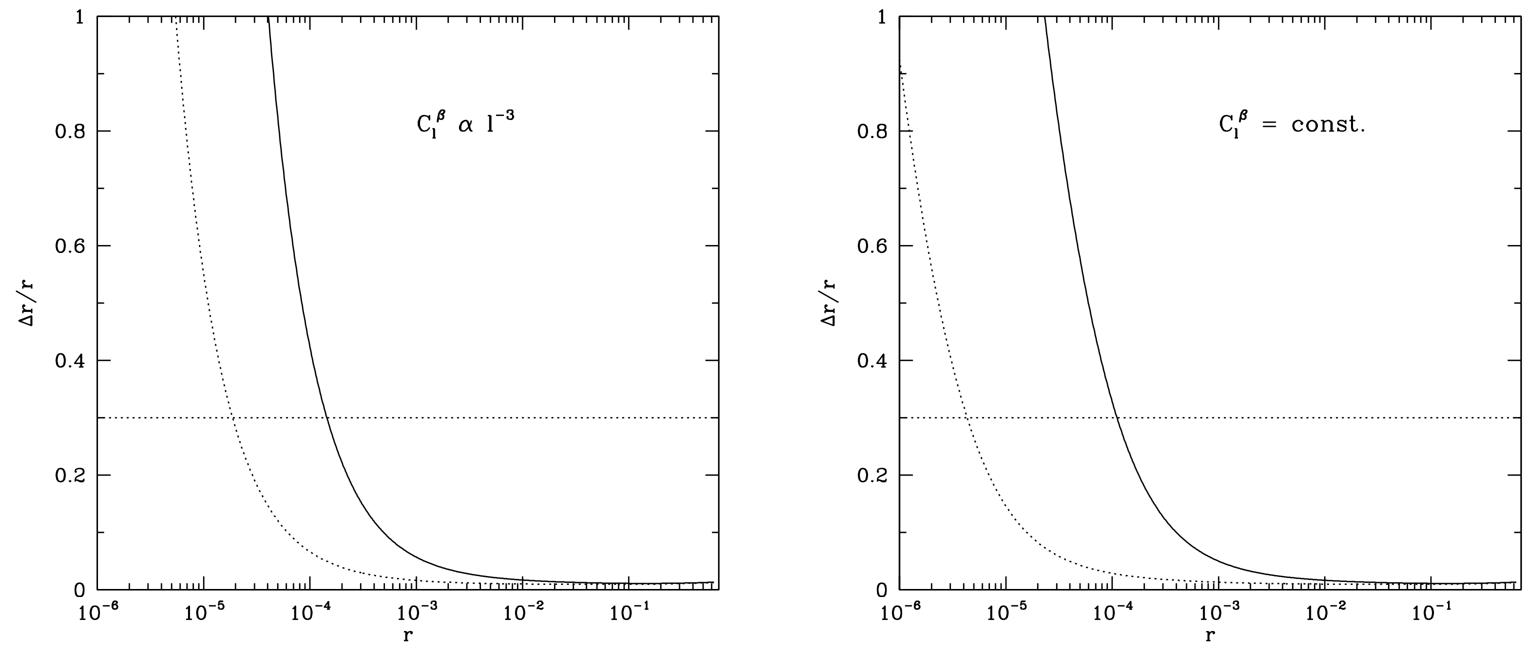
- $r_{lim} \quad \Longrightarrow \quad \frac{\Delta r}{r} = 0.3$

Constraints on r by Galactic foregrounds



Reference experiment $\Longrightarrow$ $r_{lim} = 2 \times 10^{-5}$ / 4×10^{-6}

Galactic foregrounds and Extragalactic Radio Sources



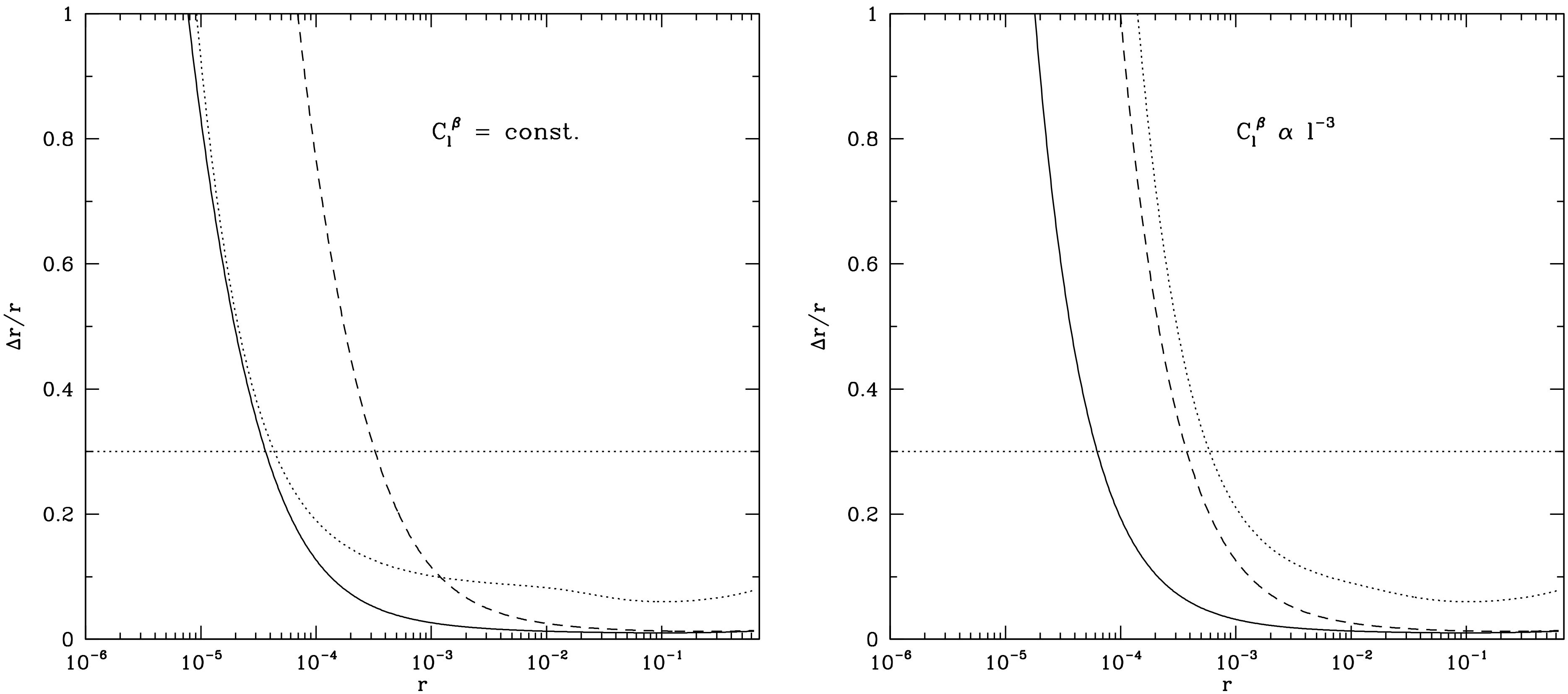
All the pixels with sources with $S > 200\,\mathrm{mJy}$ are masked ($\sim 10\%$ of 1^o–pixels)

$$r_{lim} \simeq 10^{-4}$$

Galactic foregrounds and gravitational lensing

In order to remove polarization induced by gravitational lensing, high–resolution data are required (e.g., Hu & Okamoto 2002; Seljak & Hirata 2003).

- Low–resolution experiment, no subtraction of lensing–induced polarization
- Low–resolution experiment, subtraction using information from other experiments
- High–resolution experiment (over partial sky, i.e. $30^{\circ} \times 30^{\circ}$)



– – – no subtraction $\Longrightarrow$ $r_{lim} \simeq 4 \times 10^{-4}$

——— $C_{B\ell}^{\rm lensing}/10$ $\Longrightarrow$ $r_{lim} \simeq 5 \times 10^{-5}$

.... no lensing–induced polarization $\Longrightarrow$ $r_{lim} \simeq 6 \times 10^{-4}$ *or* $\simeq 5 \times 10^{-5}$

(Area$= 30^o \times 30^o$)

Limits on r including all components

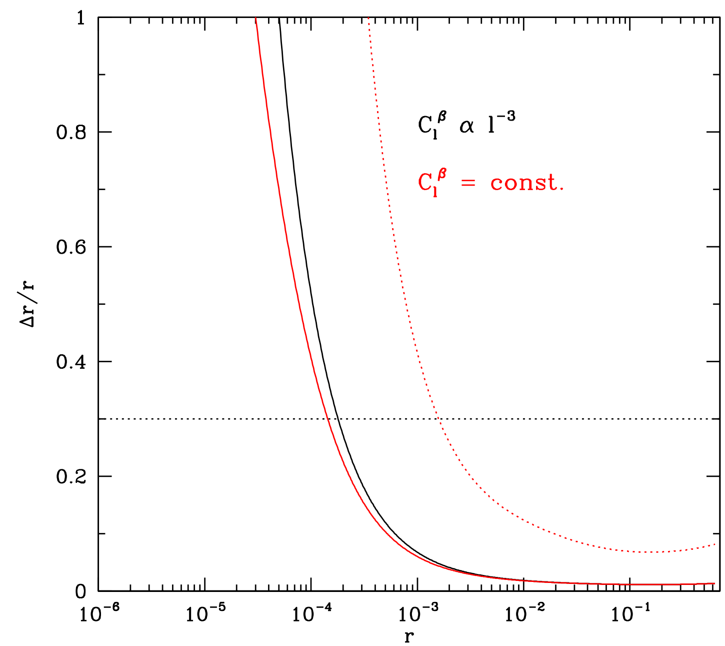
Reference experiment

- residual Galactic foregrounds
- radio sources with $S < S_c = 200\,\mathrm{mJy}$
- $C_{B\ell}^{\mathrm{lensing}}/10$

$r_{lim} \simeq 2 \times 10^{-4}$ (independently of C_ℓ^β)

Low–resolution (FWHM$= 7^\mathrm{o}$)

$r_{lim} \sim 10^{-3}$



High–resolution, Area$= 30^\mathrm{o} \times 30^\mathrm{o}$

- $C_\ell^\beta \propto \ell^{-3}$: r_{lim} always worse
- C_ℓ^β =const:

$r_{lim} = 2 \times 10^{-4}$ if no lensing and $S_c = 25\,\mathrm{mJy}$

Conclusions

We have investigated the limits that foregrounds impose on the detectability of CMB B–mode polarization. **Free–noise experiments** are considered.

The next step will be to take into account real experiments (in progress). With Planck experiment $r < 0.08$ at the 2–σ level (for WMAP $r < 0.7$)

- Multifrequency observations allow us to remove efficiently **Galactic foregrounds**. Spectral information pixel by pixel permits a much better foreground subtraction, expecially at large scales.

- **Extragalactic foregrounds** require high–resolution data for an accurate removal. They strongly constrain the detection of CMB B–mode polarization: $r_{lim} \sim 10^{-4}$ if extragalactic foregrounds are partially subtracted ($S_c = 200$mJy, $C_\ell^{lens}/10$).

- **High–resolution small–area experiments give weaker limits on r respect to full–sky experiments**

- $r_{lim} < 10^{-4}$ are possible if radio sources with $S < 200\,$mJy **are subtracted** (not masked) directly **in polarization maps** (work in progress).