Imaging the Topology of the Universe

Dmitri Pogosyan, U of Alberta

With:

Tarun Souradeep

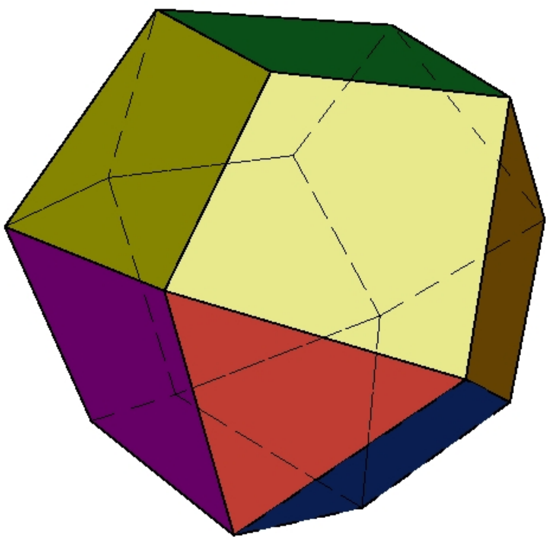
Dick Bond

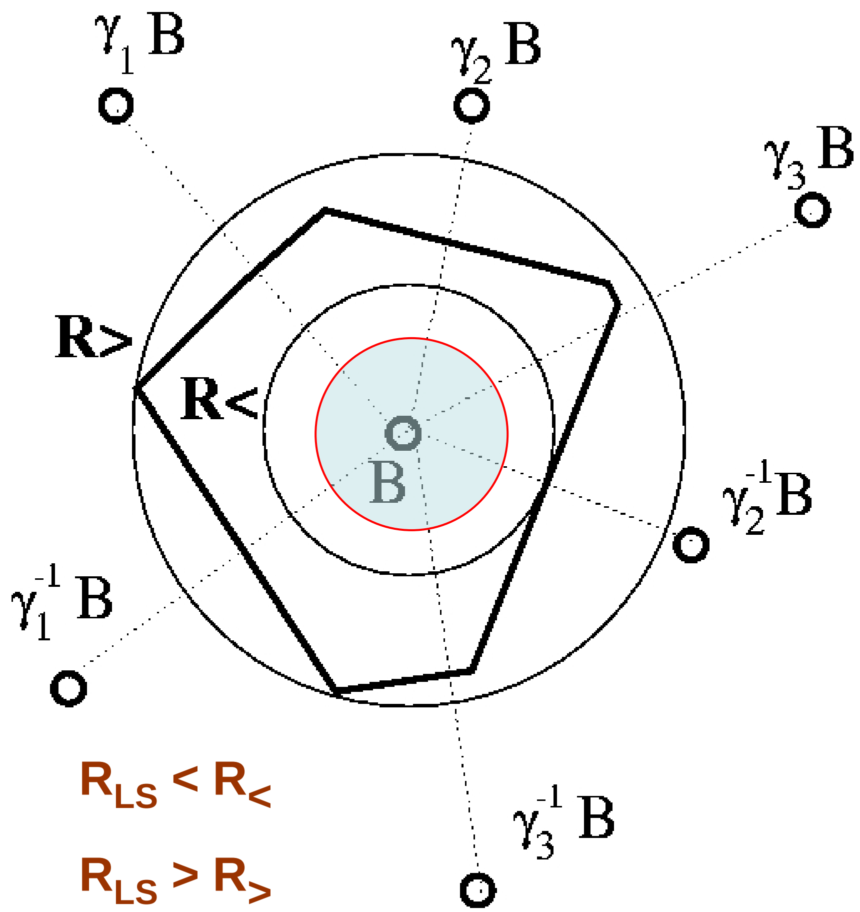
and:

Carlo Contaldi

Adam Hincks

Dirichlet domain and dimensions of the compact space

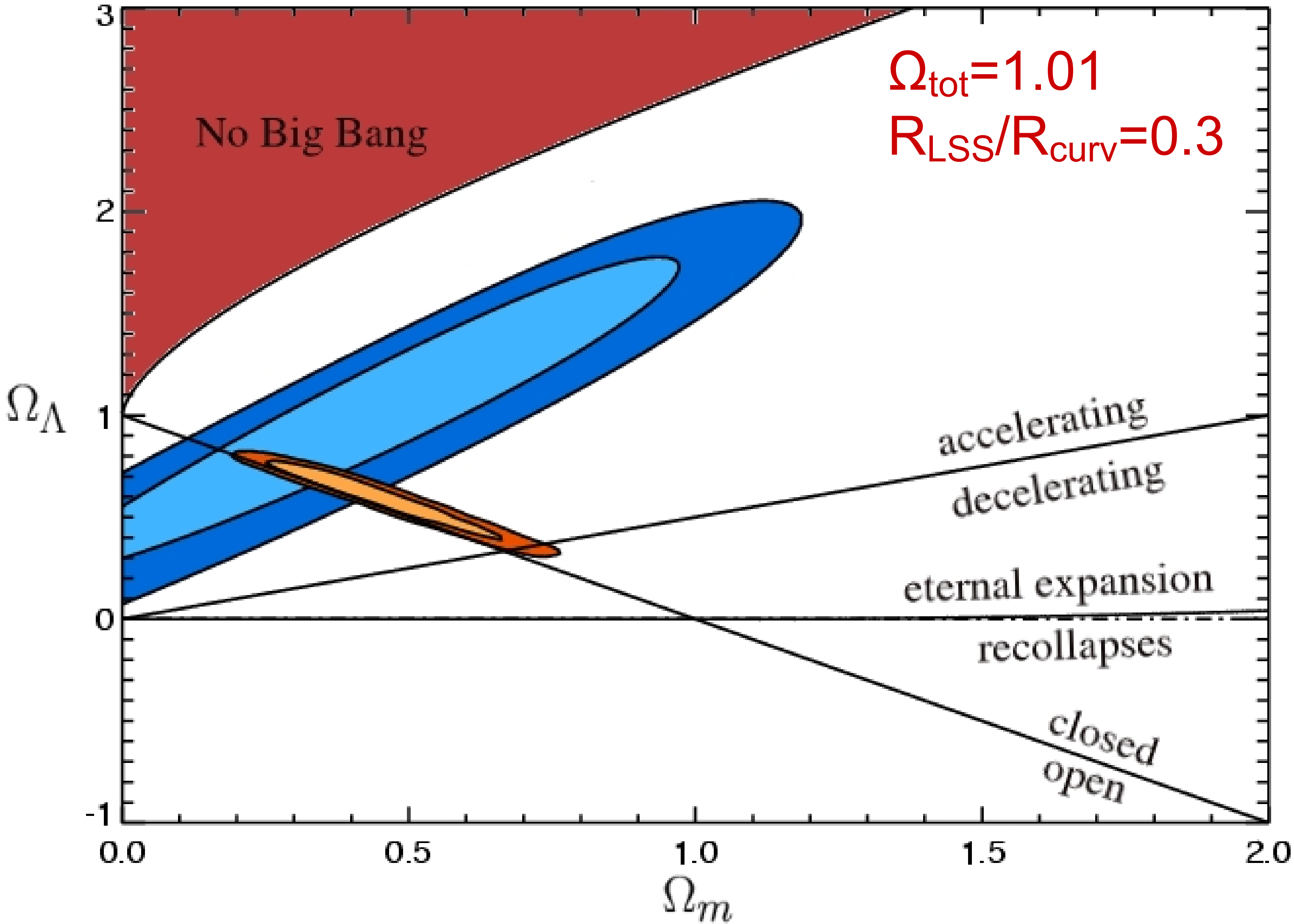




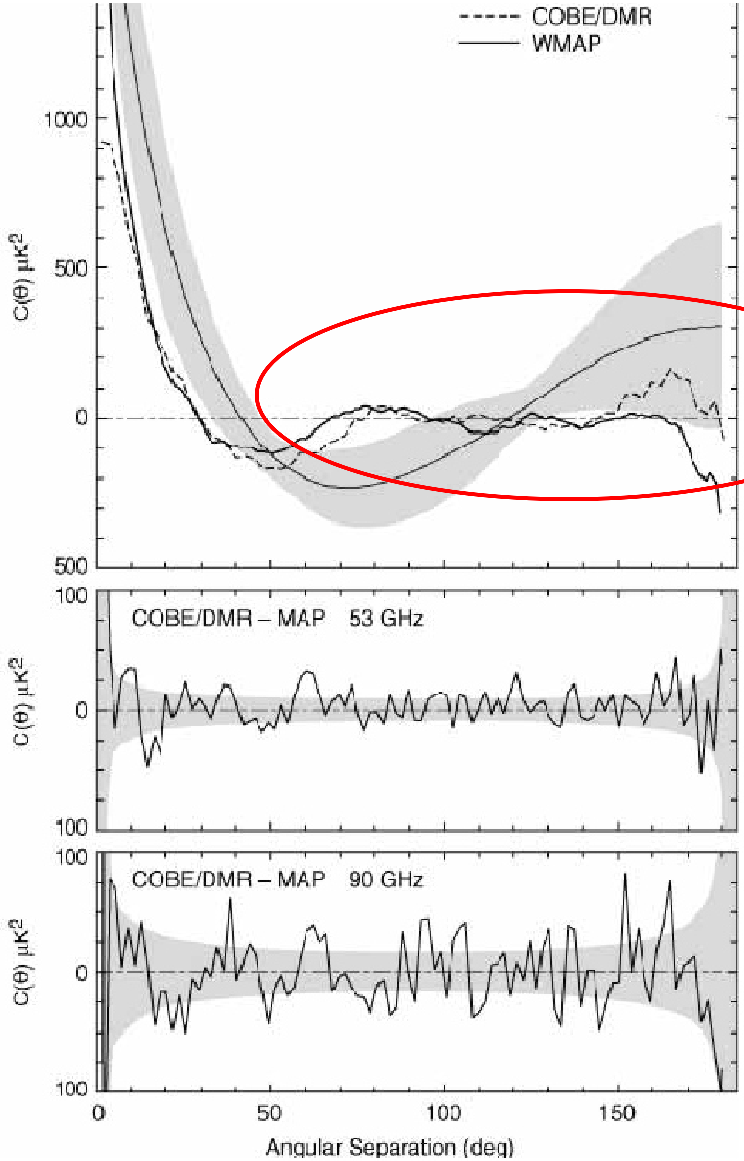
$R_{LS} < R_<$

$R_{LS} > R_>$

Preferred parameter region, Ω_{tot}=1.02 ± 0.02



Absence of large scale power

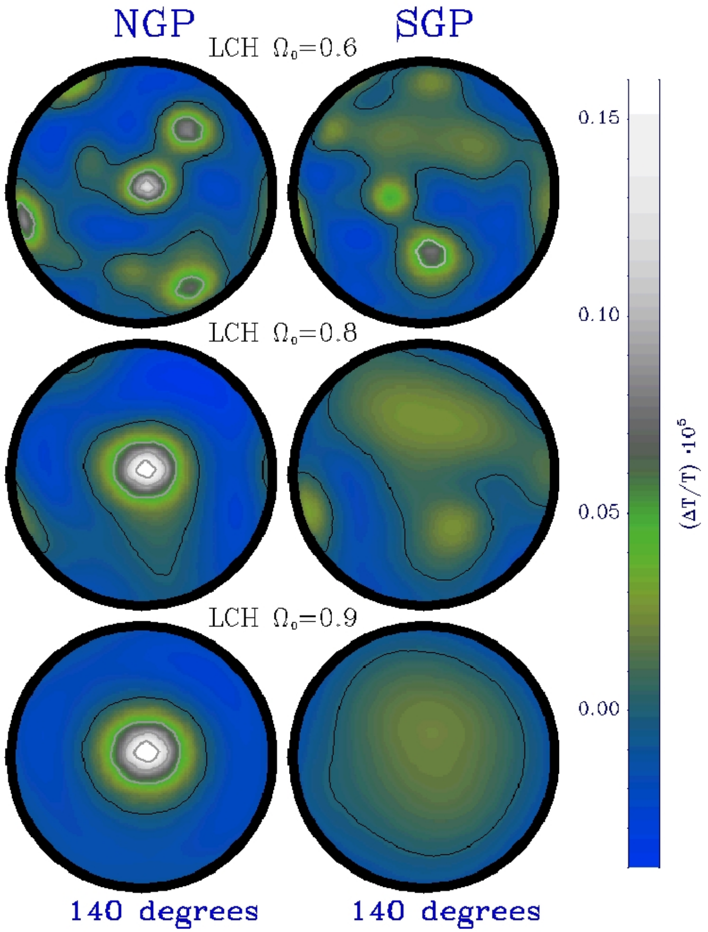


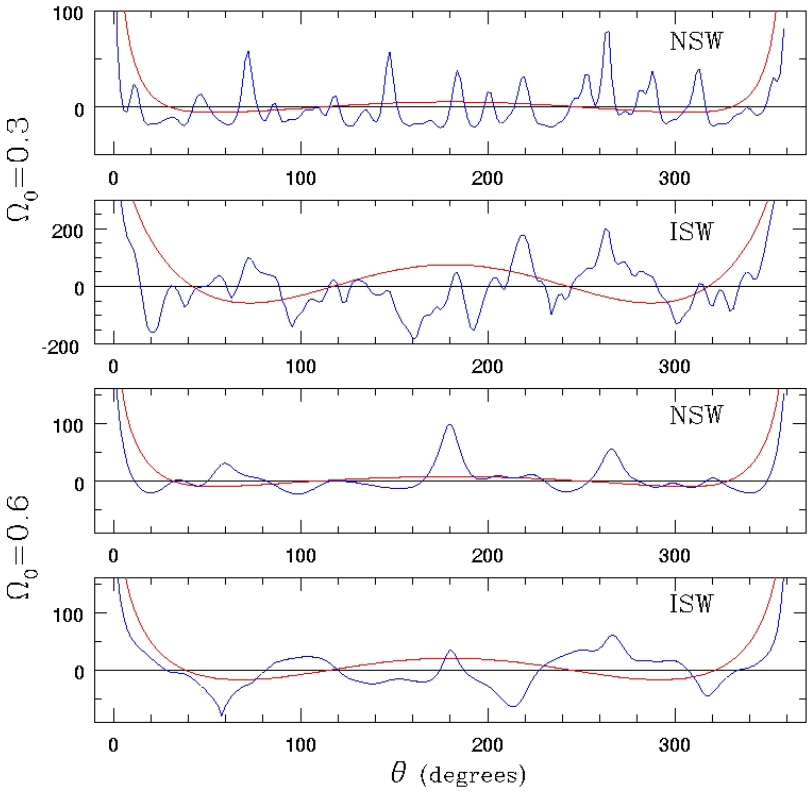
NASA/WMAP science team

Perturbations in Compact space

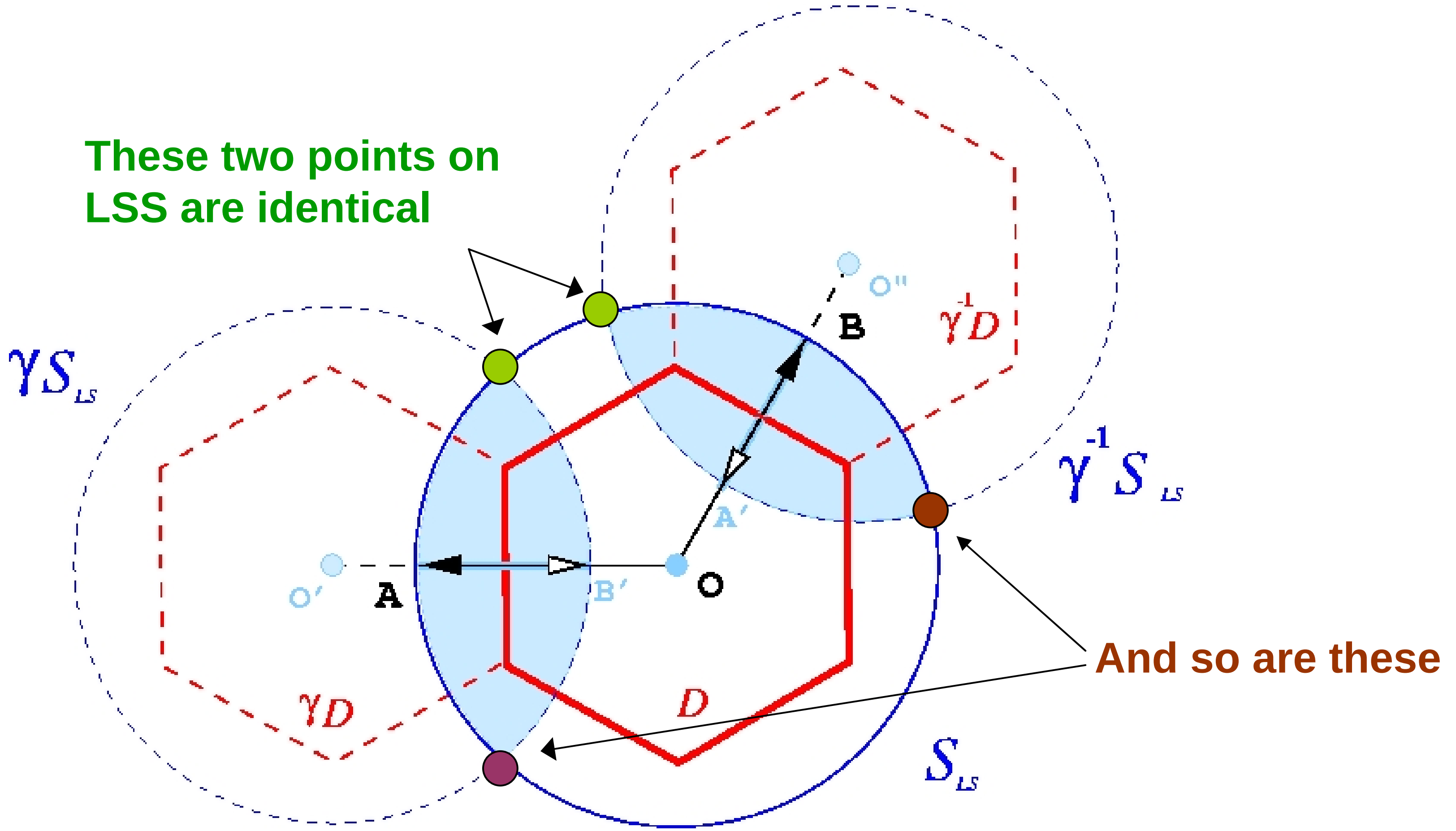
- Spectrum has the lowest eigenvalue.
- Spectrum is discrete, hence statistics in general is anisotropic, especially at large scales.
- Statistical properties can be inhomogeneous.
- However, perturbations are Gaussian, and CMB temperature fluctuations are fully described by pixel-pixel correlation matrix $C_T(p,p')$
- We implemented general method of images to compute $C_T(p,p')$ for arbitrary compact topology (Bond,Pogosyan, Souradeep, 1998,2002)

Pixel-pixel correlation with compact topology
(using method of images, BPS, Phys Rev D. 2000)



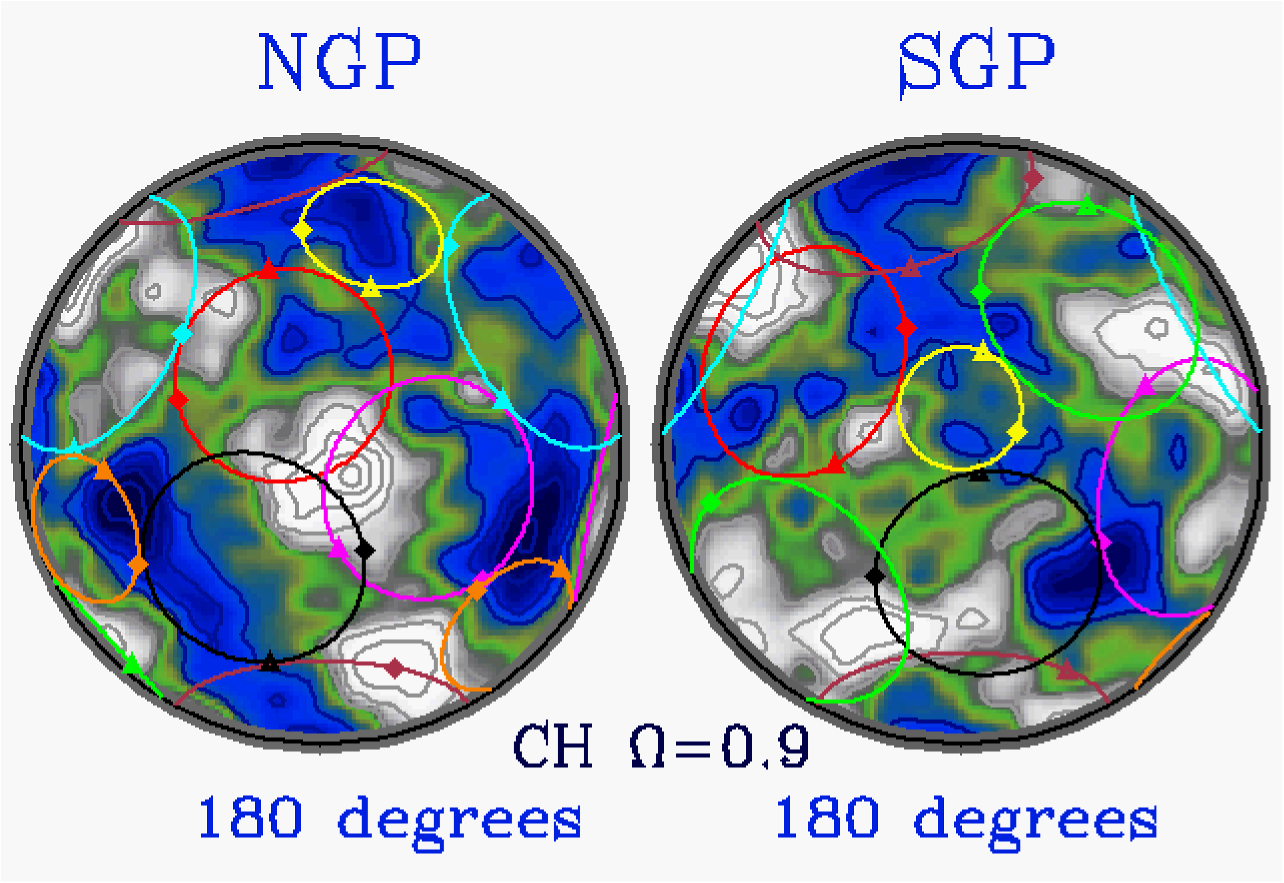


Example of strong correlation on last scattering surface



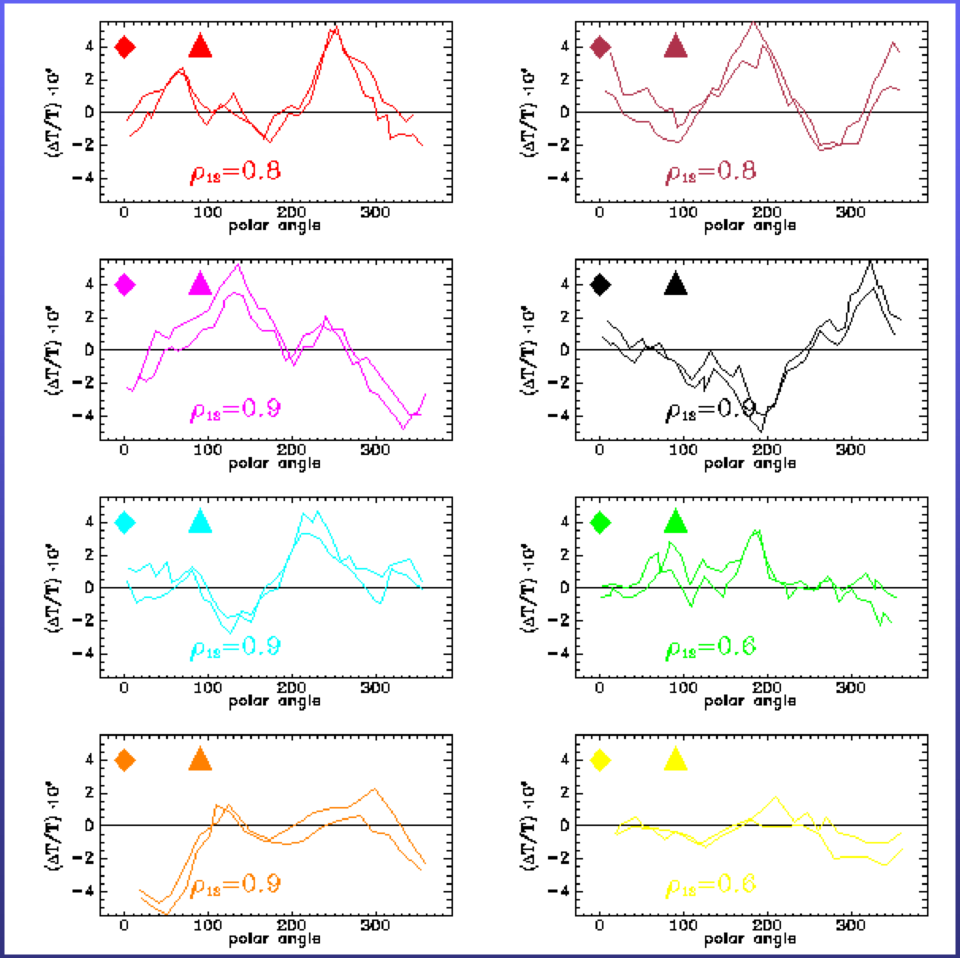
Correlated Circles

(after Cornish, Spergel, Starkman et al, 1998,2004)

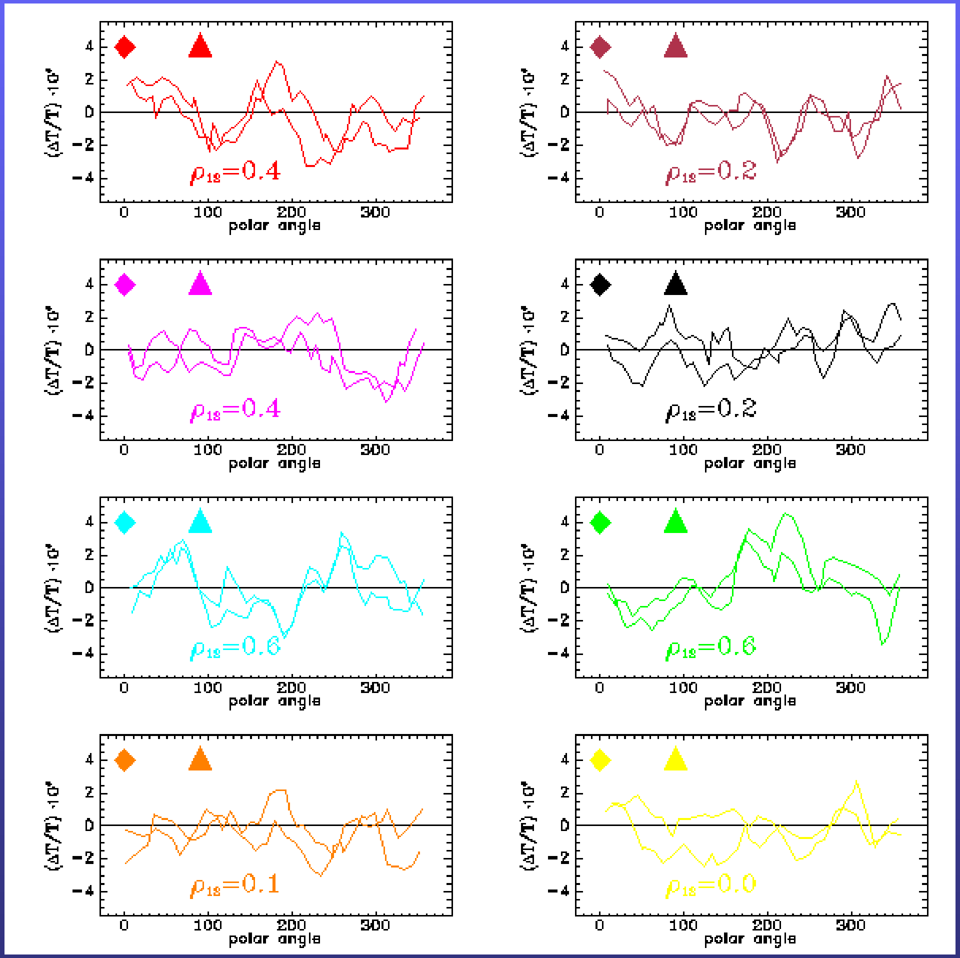


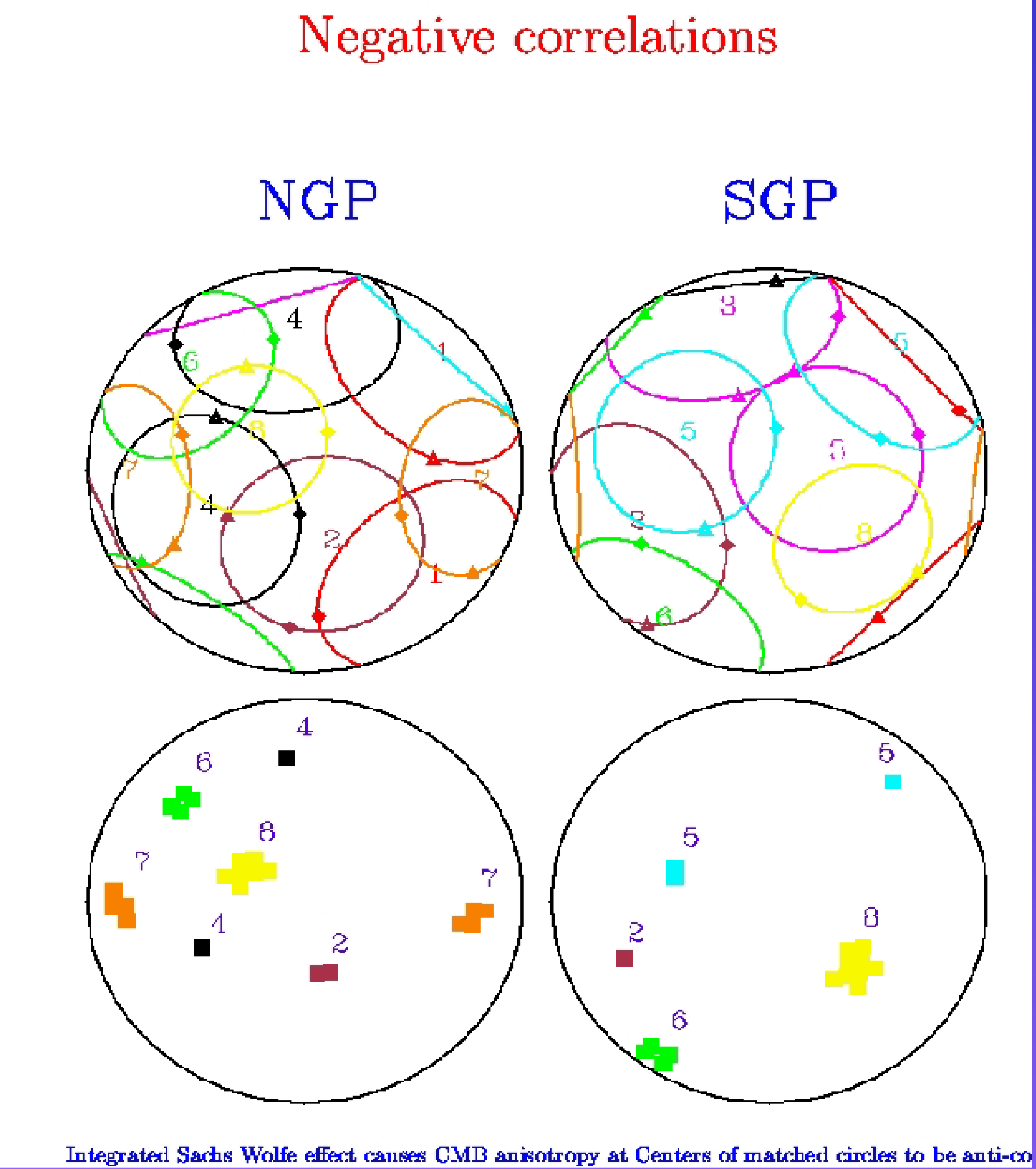
(Figure: Bond, Pogosyan & Souradeep 1998, 2002)

Temperature along the correlated circles (pure LSS signal)



Temperature along the correlated circles (ISW modification)



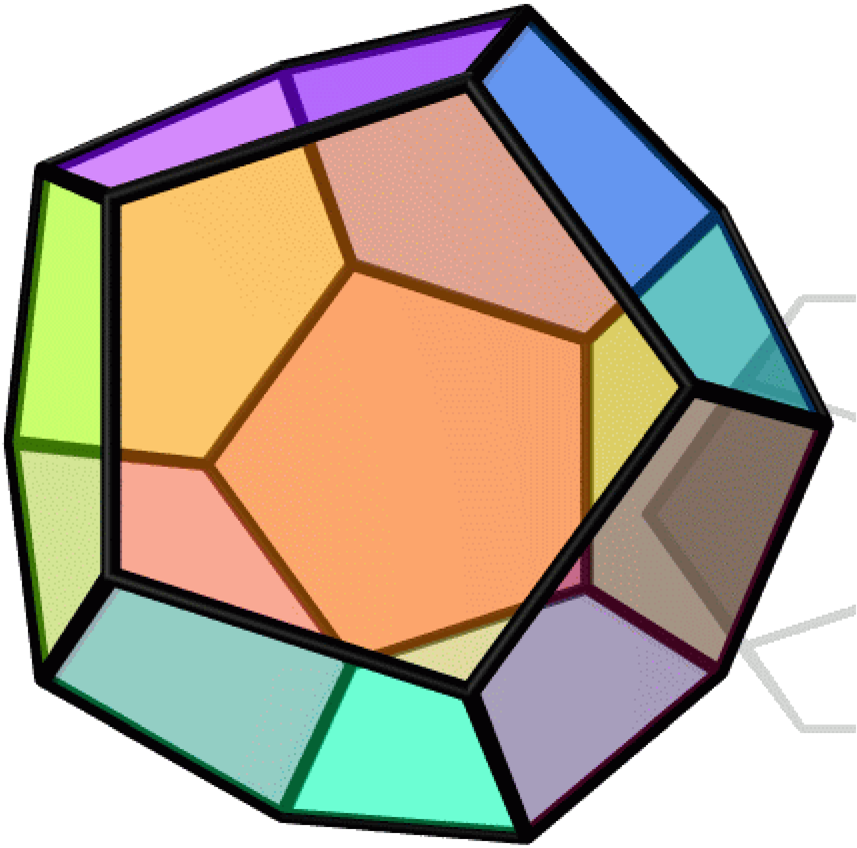

Negative correlations
NGP
SGP
4
6
8
7
4
2
1
5
3
6
Integrated Sachs Wolfe effect causes CMB anisotropy at Centers of matched circles to be anti-co

Positive curvature multiconnected universe ?

:

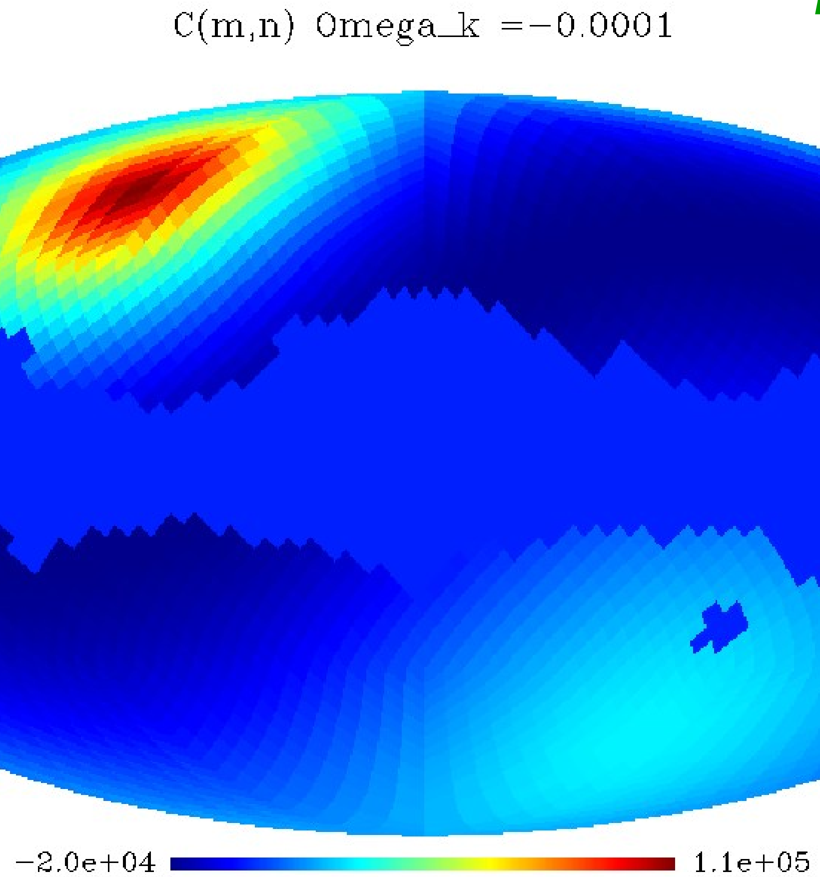
Perhaps,

a Poincare dodecahedron “Soccer ball cosmos” ?



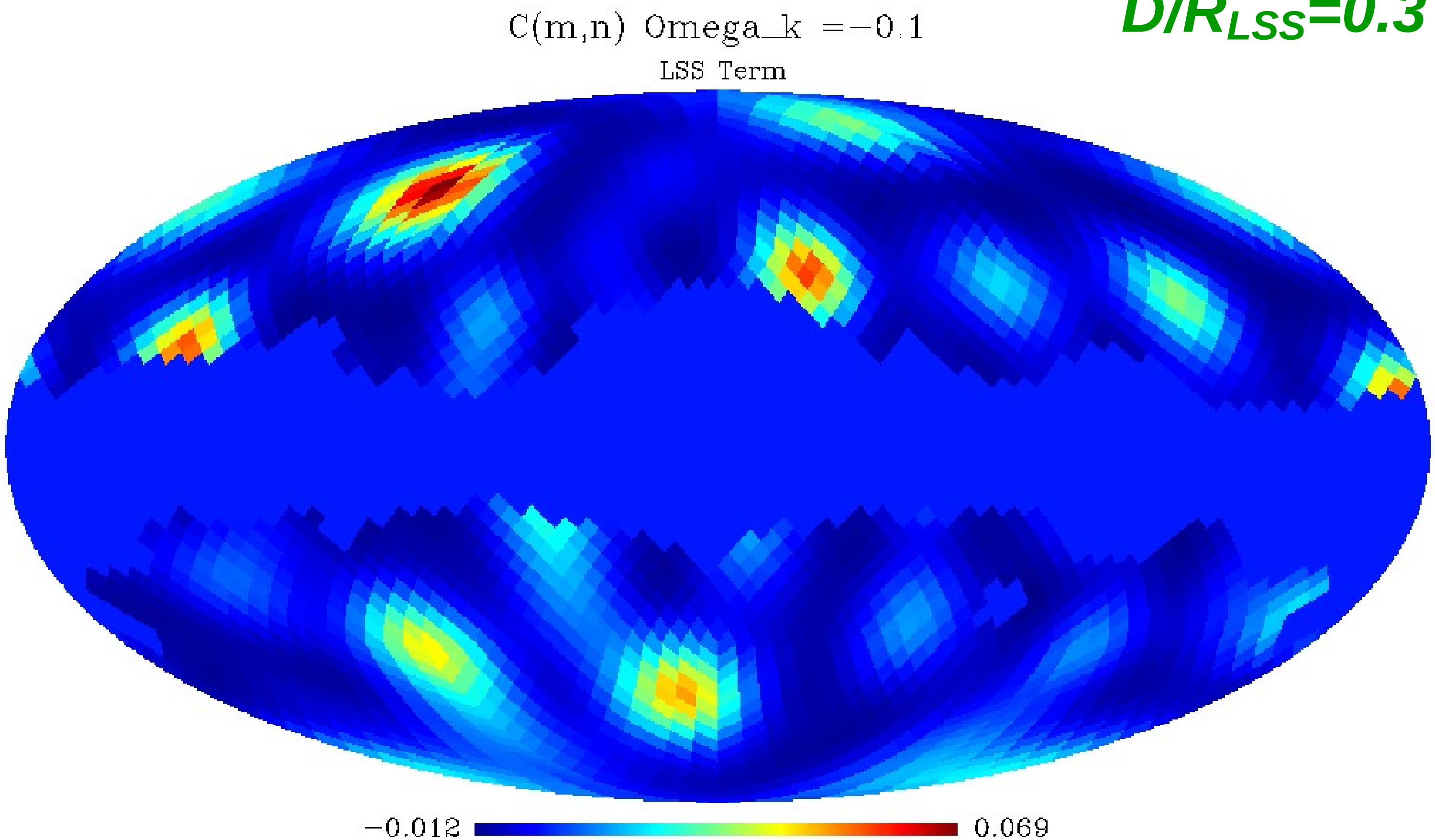
Isotropic Cpp'

D/R$_{LSS}$=10

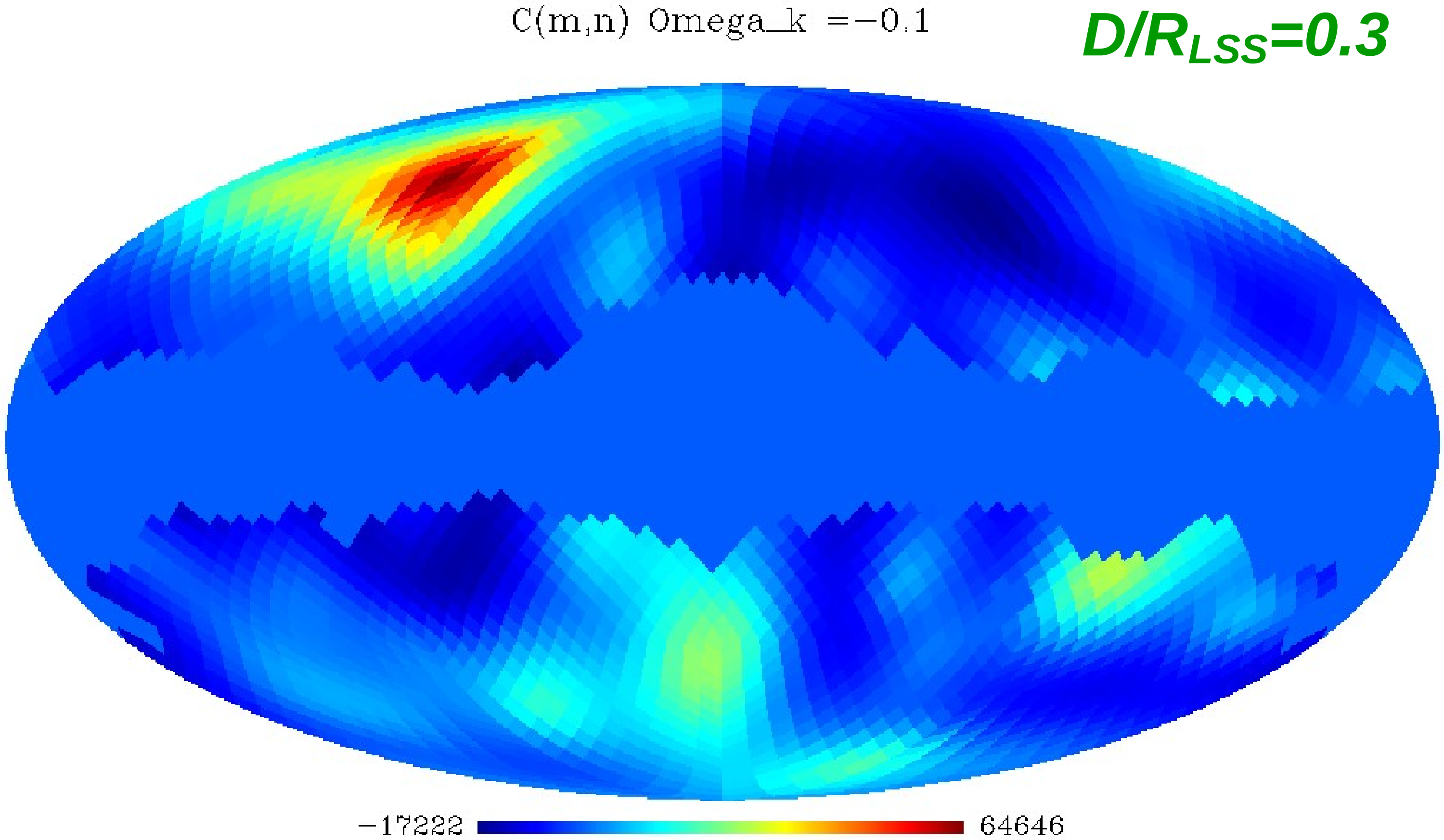


Surface term to DT/T

D/R_{LSS}=0.3



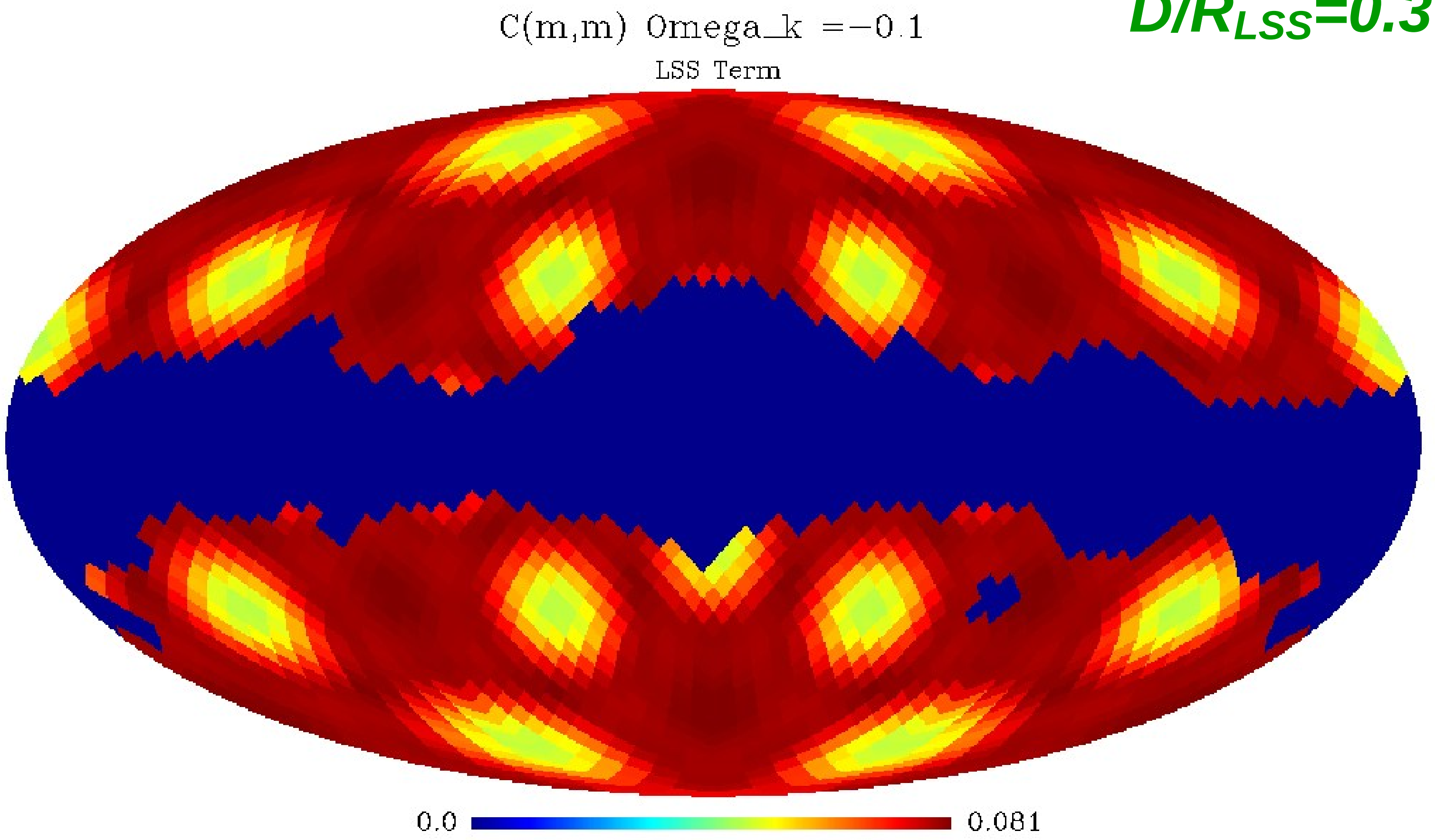
Complete, surface and integrated large-angle ΔT/T



D/R$_{LSS}$=0.3

Variation of the pixel variance

D/R_{LSS}=0.3



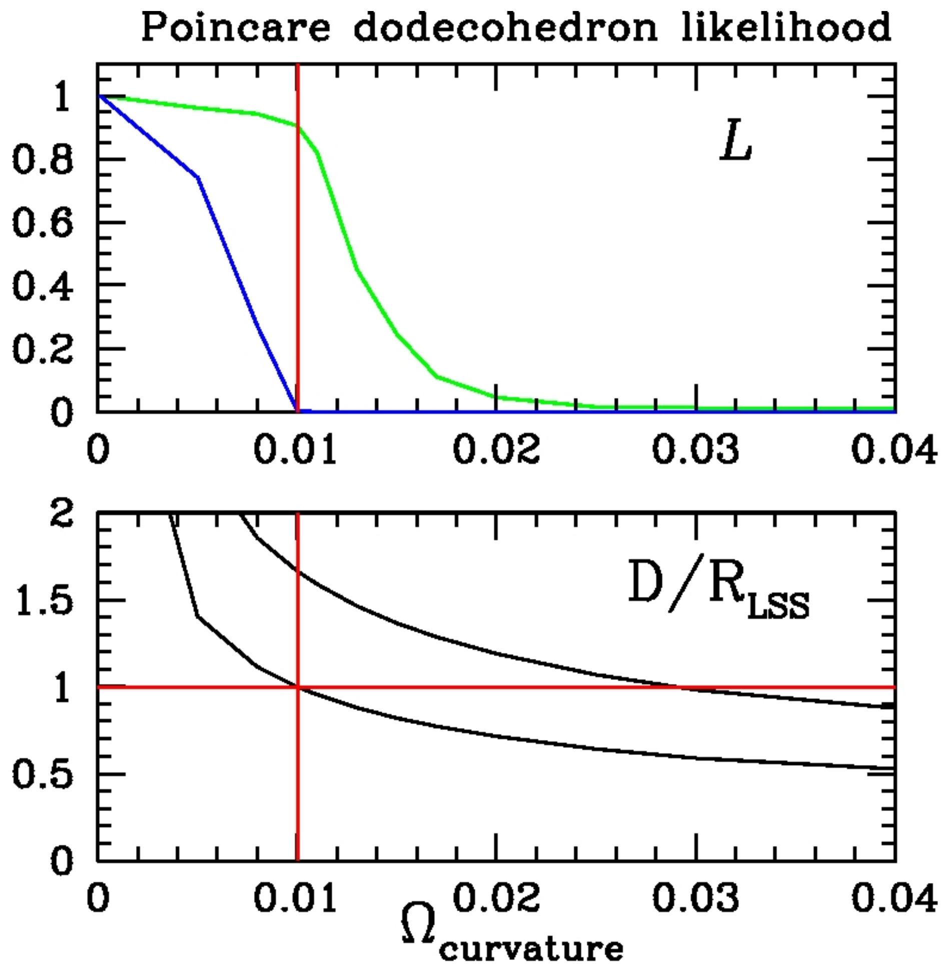
Constraining the models from maps

- **Complete topological information is retained when comparison with data is done on map level**

$$\mathcal{L}(C_T) = \frac{1}{(2\pi)^{N_p/2}\|C_N + C_T\|^{1/2}} e^{-\frac{1}{2}\bar{\Delta}^\dagger (C_N + C_T)^{-1}\bar{\Delta}} .$$

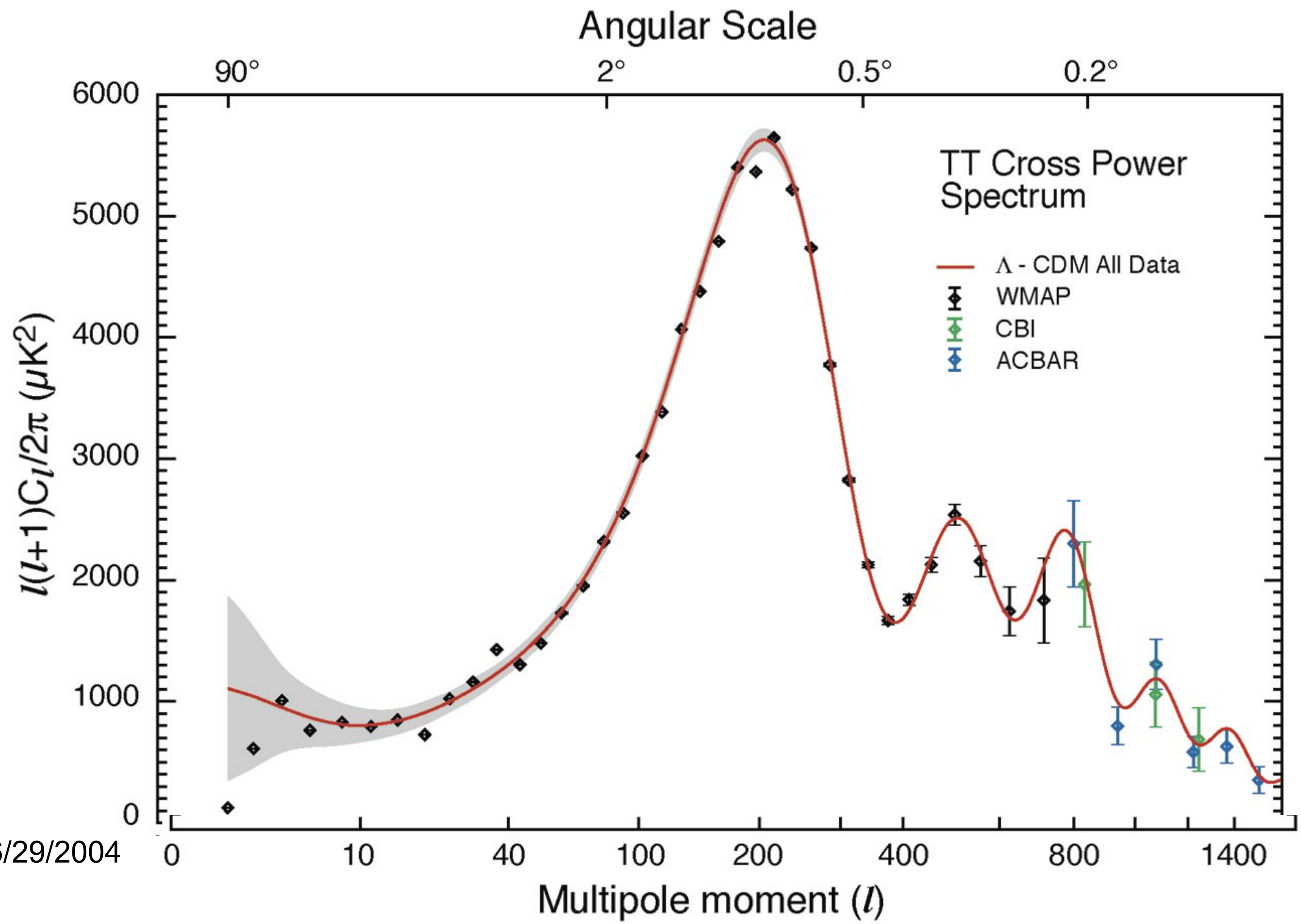
- **Low res (Nside=16) maps contain most information, although special techniques as circle searching may benefit from finer pixalization. The cost – additional small scale effects which mask topological correlations.**
- **Main signal comes from effects, localized in space, e.q on LSS. But even integrated along the line of sight contributions retain signature of compact topology.**
- **Orientation of the space (and, possibly, position of observer) are additional parameters to consider. What is the prior for them ?**

Likelehood comparison of compact closed versus flat Universe with $\Omega_\Lambda = 0.7$

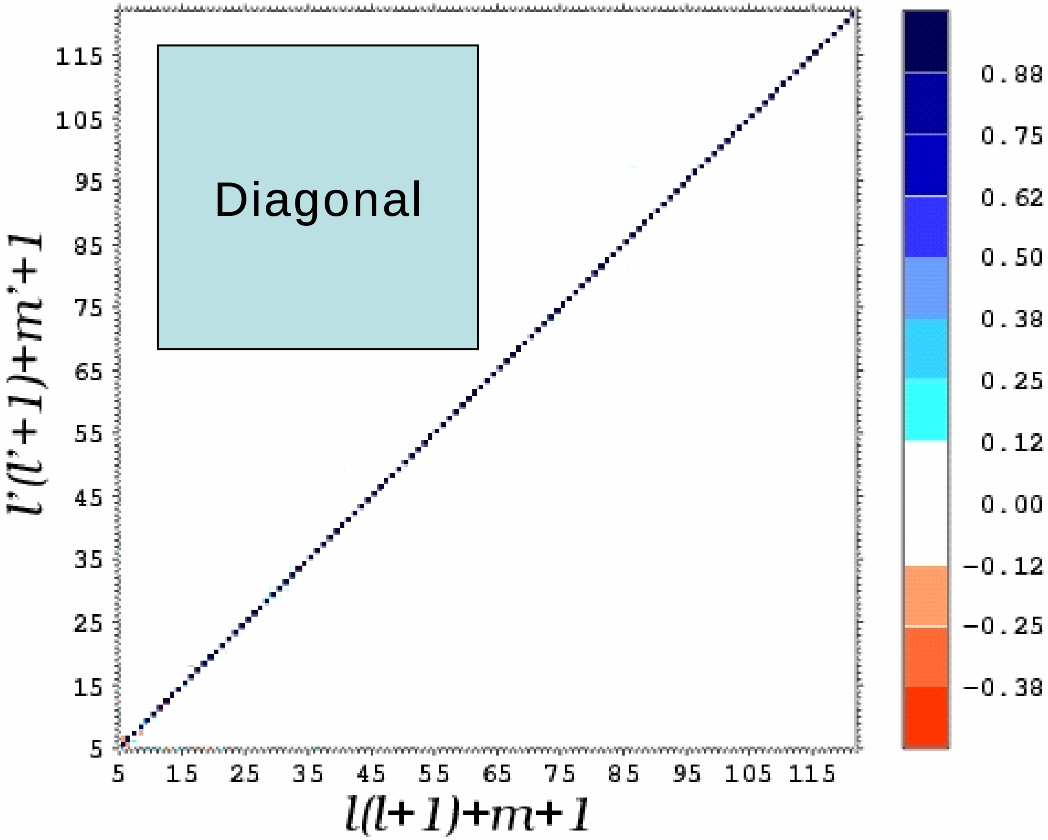


WMAP: Angular power spectrum

NASA/WMAP science team



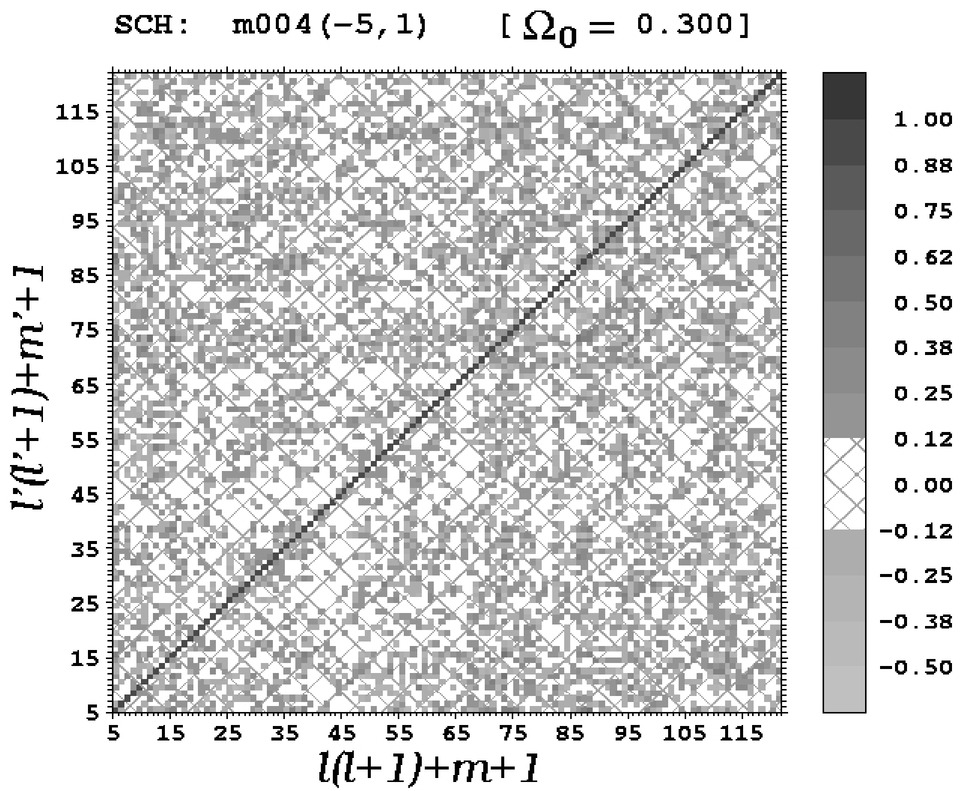
Single index n:
(l,m) -> n



$$\langle a_{lm} a^*_{l'm'} \rangle = C_l \; \delta_{ll'} \delta_{mm'}$$

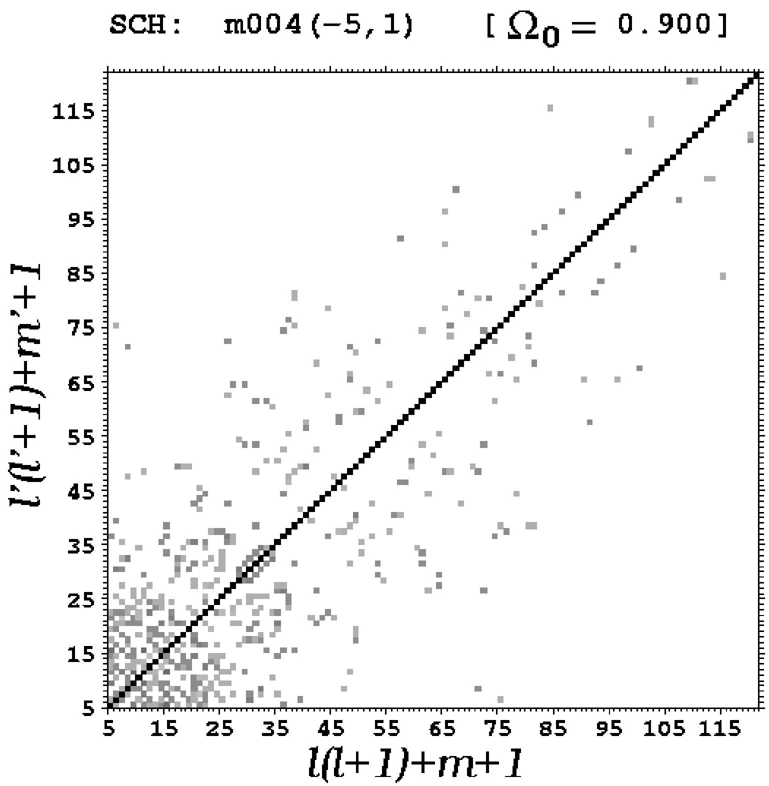
Inadequacy of isotropized C_l's: a_{lm} cross correlation

Very small space



2 3 4 5 6 7 8 9 10

Just a bit smaller than LSS

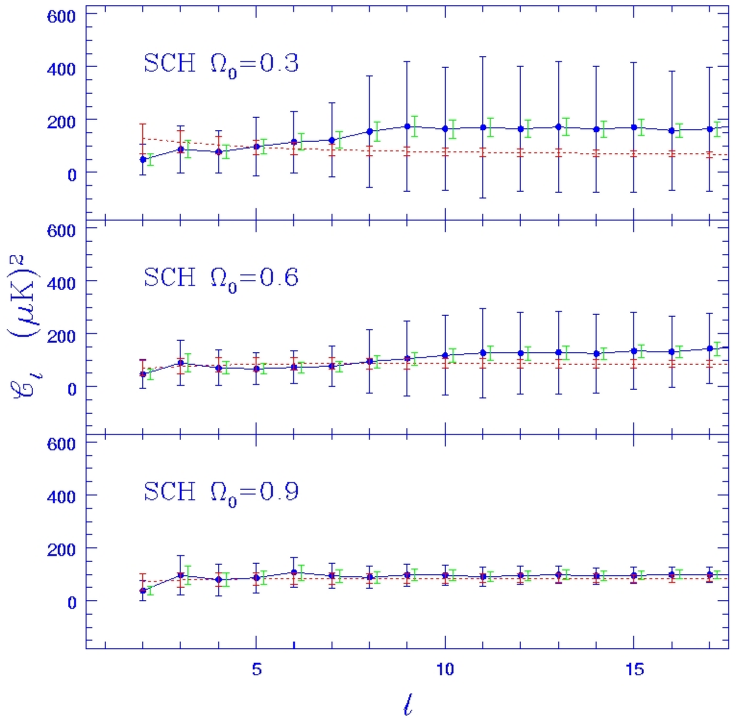


2 3 4 5 6 7 8 9 10

Compression to isotropic Cls is lossy
Inhanced cosmic variance of Cl's

$$C(\hat{q},\hat{q}') = C^{\mathrm{I}}(\hat{q},\hat{q}') + C^{\Lambda}(\hat{q},\hat{q}')$$

$$\int d\Omega_{\hat{q}} \int d\Omega_{\hat{q}'}\, C^{\Lambda}(\hat{q},\hat{q}')\, P_\ell(\hat{q}\cdot\hat{q}') = 0\,.$$



$$\langle \tilde{\mathcal{C}}_\ell \rangle = \frac{\ell(\ell+1)}{8\pi^2} \int d\Omega_{\hat{q}} \int d\Omega_{\hat{q}'}\, C(\hat{q},\hat{q}')\, P_\ell(\hat{q}\cdot\hat{q}')\,.$$

$$\mathrm{var}(\tilde{\mathcal{C}}_\ell) \equiv \langle \tilde{\mathcal{C}}_\ell^2 \rangle - \langle \tilde{\mathcal{C}}_\ell \rangle^2$$

$$\frac{2\langle \tilde{\mathcal{C}}_\ell \rangle^2}{2\ell+1} + \frac{\ell^2(\ell+1)^2}{32\pi^4} \int d\Omega_{\hat{q}_1} \int d\Omega_{\hat{q}_2} \left[\int d\Omega_{\hat{q}_3}\, C^{\Lambda}(\hat{q}_1,\hat{q}_3)\, P_\ell(\hat{q}_2\cdot\hat{q}_3)\right]^2 .$$

Bipolar Power spectrum (BiPS) :
A Generic Measure of Statistical Anisotropy

$$\kappa^{\lambda} = (2\lambda+1)^2 \int d\Omega_{n_1} \int d\Omega_{n_2} [\frac{1}{8\pi^2} \int d\Re\, \chi^{\lambda}(\Re) C(\Re\hat{n}_1, \Re\hat{n}_2)]^2$$

Bipolar multipole index

A **weighted average** of the correlation function over all rotations

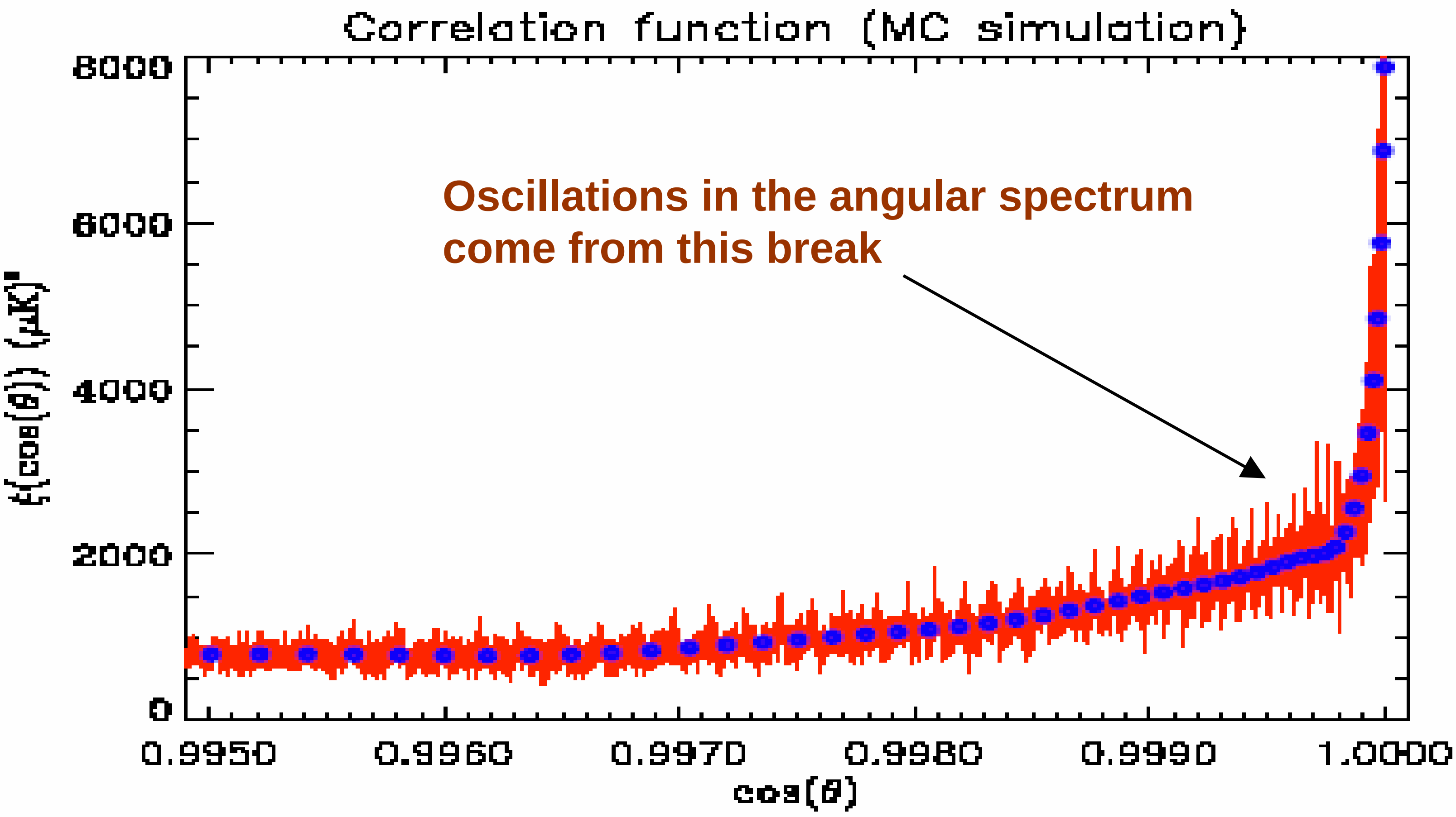
Except for $\lambda = 0$ when $\chi^{0}(\Re) = 1$

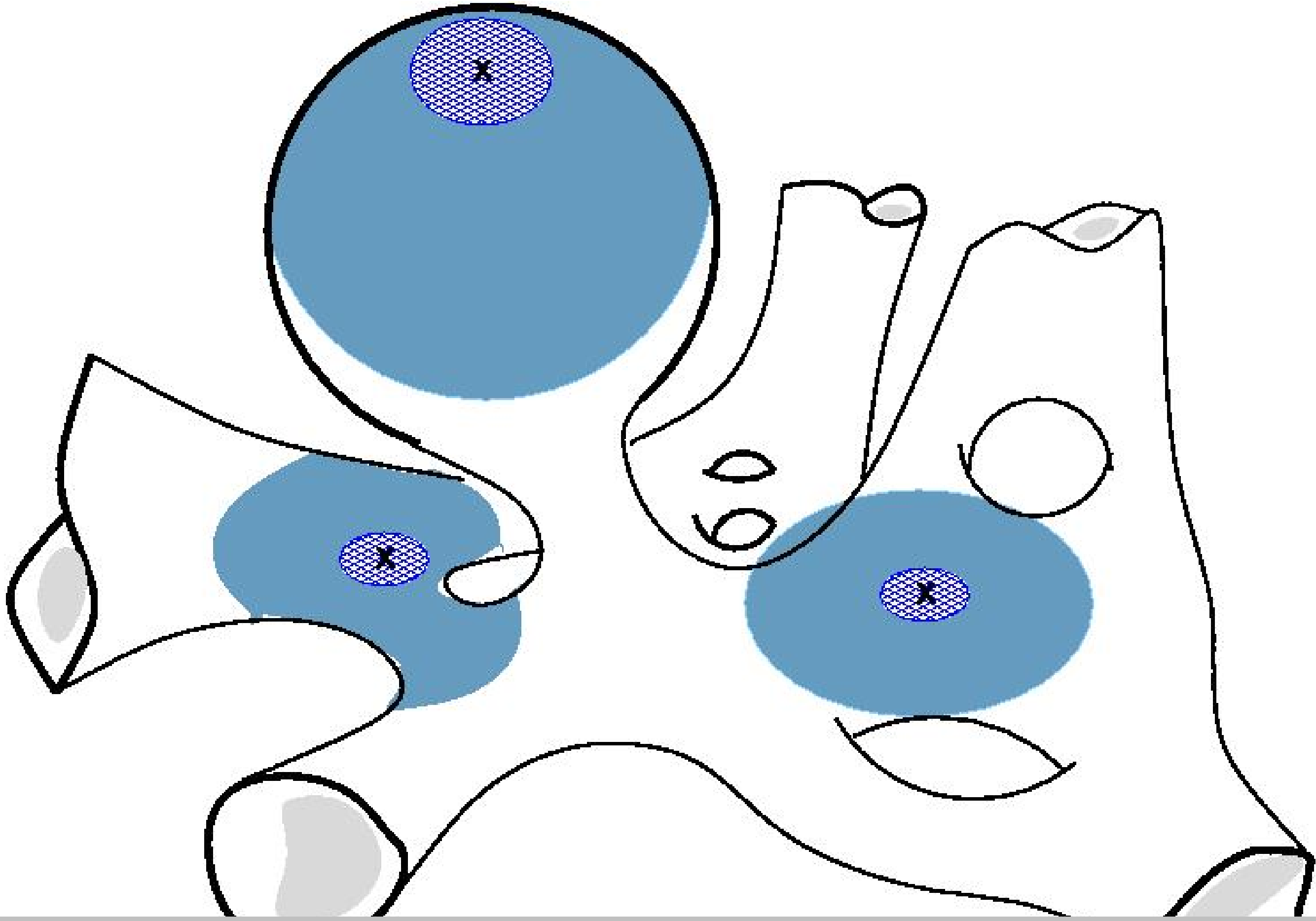
(This slide is provided as a free advertisement for T. Souradeep talk, Friday, 9.35am)

Conclusions

- The region near Otot=1 is rich with possibilities, with negatively or positively curved or flat spaces giving rise to distinct topological choices.
- Modern CMB data shows that small Universes with V < VLS are failing to describe the temperature maps. Reason – complex correlation are not really observed (in line with circle finding results).
- This is despite the fact that it is not too difficult to fit the low l suppression of isotropized angular power spectrum.
- Integrated along the line of sight contribution to temperature anisotropy masks and modifies topology signature. It must be taken into account for any accurate quantitative restrictions placed on compact models.
- Full likelihood analysis assumes knowledge of the models. Model independent search for statistical anisotropy calls for specialized techniques – see talk on BiPS by Tarun Souradeep on Thursday.

Correlation function of isotropic CMB





How Big is the Observable Universe ?

Relative to the local curvature & topological scales

Iso-contours of correlation around a point

Radical breakdown of SI

disjoint iso-contours

multiple imaging

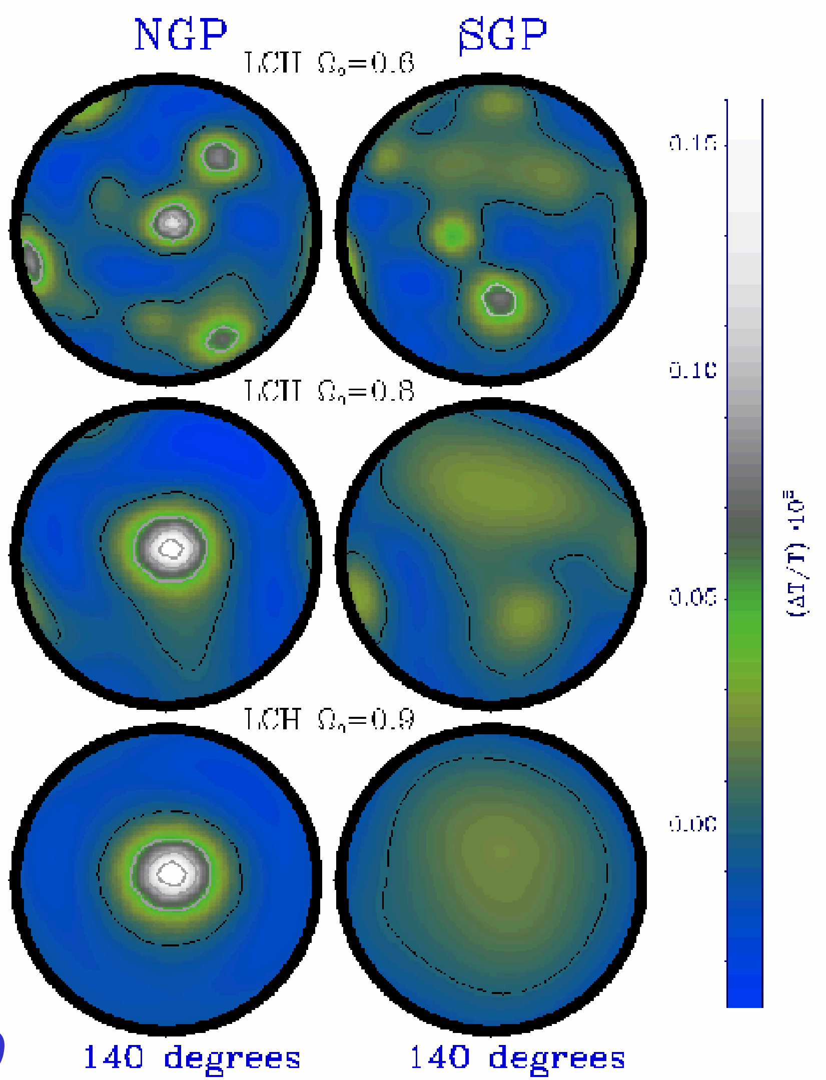
Mild breakdown of SI

Distorted iso-contours

Statistically isotropic (SI)

Circular iso-contours

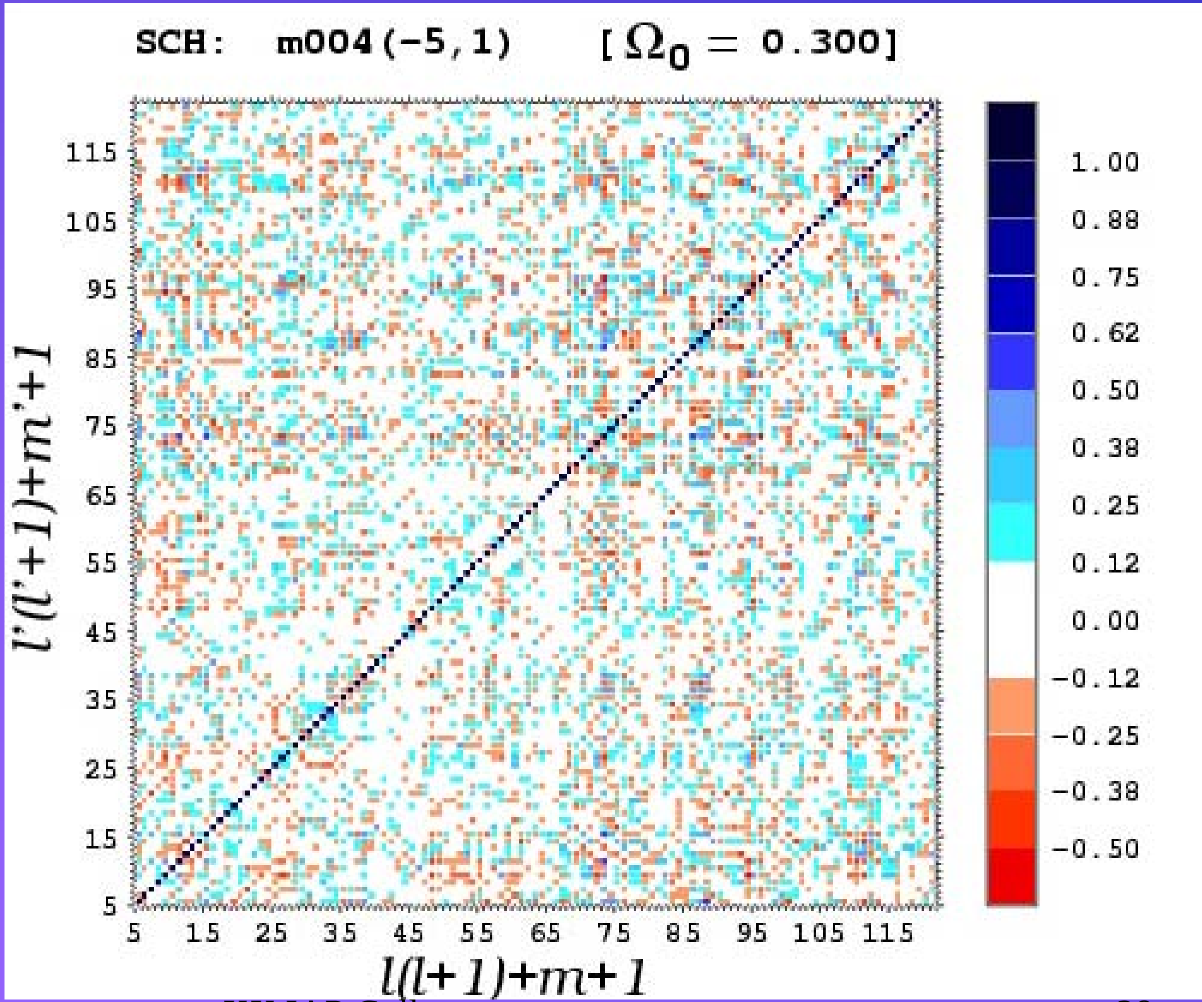
(Bond, Pogosyan & Souradeep 1998, 2002)



SI violation : $\langle a_{lm} a^*_{l'm'} \rangle \neq C_l \ \delta_{ll'} \delta_{mm'}$

Radical breakdown

$$\frac{\langle a_{lm} a^*_{l'm'} \rangle}{\sqrt{\langle a_{l'm'} a^*_{l'm'} \rangle \langle a_{lm} a^*_{lm} \rangle}}$$



(Bond, Pogosyan & Souradeep 1998, 2002)