

20TH IAP COLLOQUIUM ON  
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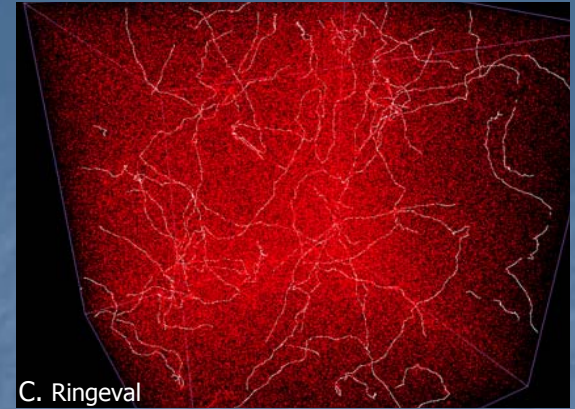
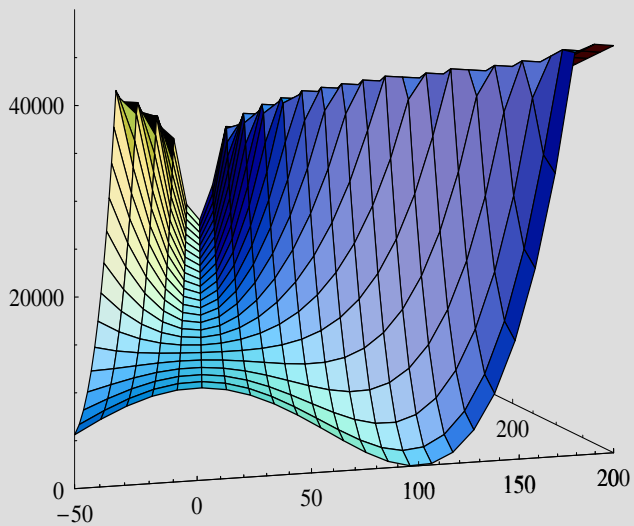
# Constraining SUSY GUT models of inflation with the CMB

by

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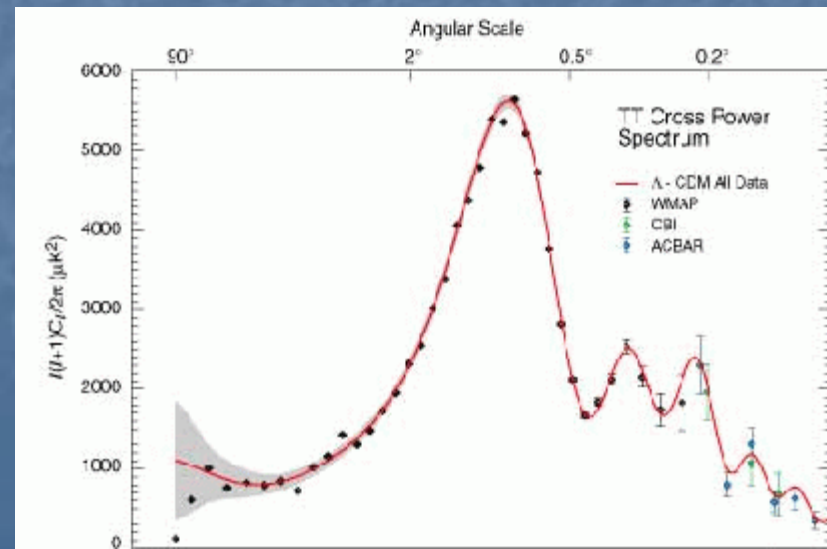
Collaboration : M. Sakellariadou

References : J. R., M. Sakellariadou (2004) [hep-ph/0405133,  
hep-ph/0406120]



## Outline

- Framework and motivations
- Contributions to CMB anisotropies
  - F-term inflation
  - D-term inflation
- Constraints on inflationary models
- Conclusions



# Framework and Motivations

[Jeannerot, J. R., Sakellariadou, PRD68 (2003)]

Framework = Supersymmetric Grand Unified Theories

GUT  $\longrightarrow$  SSBs + phase transitions  $\longrightarrow$  Topological defects.

Selection of SSB patterns of GUT groups down to the SM in agreement with :

- Particle physics (neutrino oscillation, proton lifetime, SM)
- Cosmology (CMB observations, monopole problem, matter/antimatter asymmetry)

This necessitates a phase of inflation after the formation of monopoles :

Supersymmetric Hybrid Inflation.

It couples the inflaton with a pair of GUT Higgs fields. It ends with the SSB.



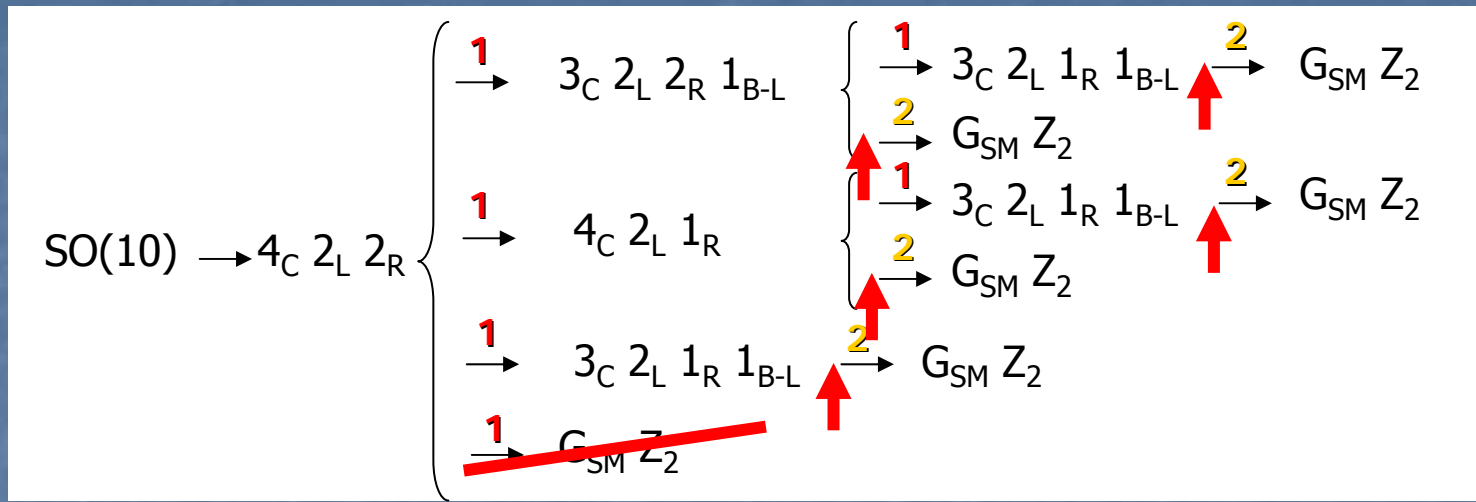
?

GUT

Inflation

Standard Model

time →



1: Monopoles    2: Cosmic Strings    ↑ INFLATION

Idem for SO(10), SU(5), SU(6), SU(7), E6, ...

Conclusions :

- In SUSY GUTs, formation of cosmic strings = **GENERIC**
- Formation at the end of inflation :  $G_{\mu} \sim M_{infl}^2$

ARE THEY CONSISTENT WITH CMB DATA ?

WHAT IS THEIR CONTRIBUTION TO THE CMB ANISOTROPIES ?

# Contribution to CMB anisotropies

## 1. Inflationary part

$$\left(\frac{\delta T}{T}\right)_{Q\text{-infl}}^2 = \left(\frac{\delta T}{T}\right)_{Q\text{-scal}}^2 + \left(\frac{\delta T}{T}\right)_{Q\text{-tens}}^2$$

Sachs-Wolfe effect :



$$\left(\frac{\delta T}{T}\right)_{Q\text{-scal}} \propto \frac{V^{3/2}(\varphi_Q)}{M_{\text{P}}^3 V'(\varphi_Q)}$$

$$\left(\frac{\delta T}{T}\right)_{Q\text{-tens}} \propto \frac{V^{1/2}(\varphi_Q)}{M_{\text{P}}^2}$$

Number of e-foldings of inflation :

$$N_Q \equiv N(\varphi_Q \rightarrow \varphi_{\text{end}}) = -\frac{1}{M_{\text{P}}^2} \int_{\varphi_Q}^{\varphi_{\text{end}}} \frac{V(\varphi)}{V'(\varphi)} d\varphi$$

To solve the horizon problem,  
we need at least  $N_Q \sim 60$ .

# The $\mathcal{F}$ term inflation

Superpotential :  $W^F = \kappa S(\Phi_+ \Phi_- - M^2)$

S  
U  
S  
Y



$$V^F(\phi_+, \phi_-, S) = \kappa^2 |M^2 - \phi_+ \phi_-|^2 + \kappa^2 |S|^2 (|\phi_-|^2 + |\phi_+|^2) + D\text{-terms}$$

[Dvali, Shafi, Schaefer PRL73 (1994)]

A global minimum for  $\langle \phi_+ \rangle = \langle \phi_- \rangle = M$  and  $\langle S \rangle = 0$  and a local minimum for  $S \gg S_C$ ,  $\langle \phi_+ \rangle = \langle \phi_- \rangle = 0$ ,  $V_0 = \kappa^2 M^4$ .

→ Perfectly flat direction + F-term SUSY breaking => mass splitting of components of  $\Phi$ .



1-loop radiative corrections [Coleman and Weinberg, PRD7 (1973)]

$$V_{\text{eff}}(S) = \kappa^2 M^4 \left\{ 1 + \frac{\kappa^2 N}{32\pi^2} \left[ 2 \ln \frac{|S|^2 \kappa^2}{\Lambda^2} + (z+1)^2 \ln(1+z^{-1}) + (z-1)^2 \ln(1-z^{-1}) \right] \right\}$$

where

$$z = \frac{|S|^2}{M^2} \equiv x^2$$

Assuming  $\Delta V \ll V_0$ , and using the complete effective potential for  $V'(|S|)$ ,

$$\left(\frac{\delta T}{T}\right)_{Q\text{-scal}} \approx \frac{1}{\sqrt{45}} \sqrt{\frac{N_Q}{N} \frac{M^2}{M_P^2}} \left[ x_Q y_Q f(x_Q^2) \right]^{-1}$$

[Senoguz and Shafi,  
PLB567 (2003)]

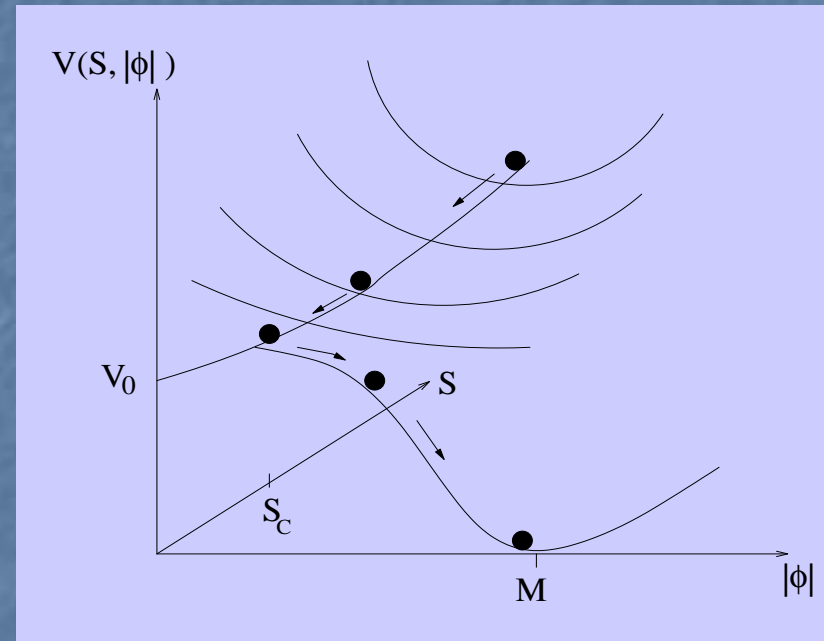
$$\left(\frac{\delta T}{T}\right)_{Q\text{-tens}} \approx \frac{0.77}{8\pi} \kappa \frac{M^2}{M_P^2}$$

where

$$y_Q^2 = \int_1^{x_Q^2} \frac{dz}{zf(z)}$$

and

$$f(z) = (z+1) \ln(1+z^{-1}) + (z-1) \ln(1-z^{-1})$$



Calculating the number of e-fold  $N_Q$ , we obtain

$$\frac{M}{M_P} = \frac{\kappa \sqrt{N N_Q}}{2\pi} y_Q^{-1}$$

# Contribution to CMB anisotropies

## 2. Cosmic strings part

The cosmic strings contribution is proportionnal to the mass per unit length  $\mu$ . For a network of Nambu-Goto strings,

$$\left(\frac{\delta T}{T}\right)_{Q\text{-strings}} \sim (9-10)G\mu \quad \text{where} \quad \mu = 2\pi\langle h \rangle^2$$

[Landriau and Shellard, astro-ph/0302166]

where,  $\langle h \rangle$  is the VEV of the higgs field responsible of the strings formation. Here,

$$\langle h \rangle = \langle \phi_{\pm} \rangle = M \quad \text{and} \quad \left(\frac{\delta T}{T}\right)_{Q\text{-strings}} \approx \frac{9}{4} \frac{M^2}{M_{\text{P}}^2}$$



# Contribution to CMB anisotropies

## 3. Total part

In conclusion, three sources contribute to the CMB quadrupole anisotropy :

$$\left(\frac{\delta T}{T}\right)_{Q\text{-tot}}^2 = \left(\frac{\delta T}{T}\right)_{Q\text{-scal}}^2 + \left(\frac{\delta T}{T}\right)_{Q\text{-tens}}^2 + \left(\frac{\delta T}{T}\right)_{Q\text{-strings}}^2$$

where the r.h.s. must be normalised to COBE  $(\delta T/T)_Q \sim 6 \times 10^{-6}$ .

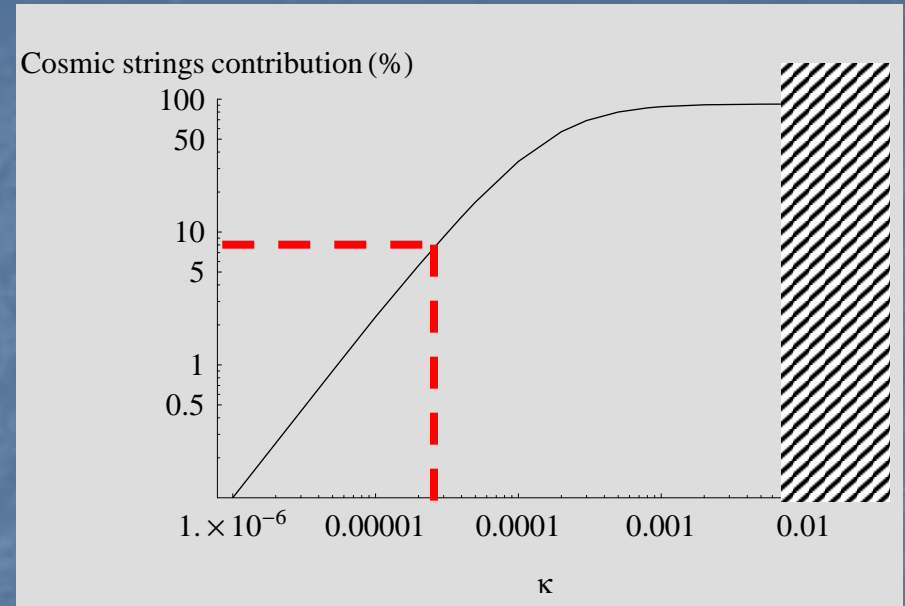
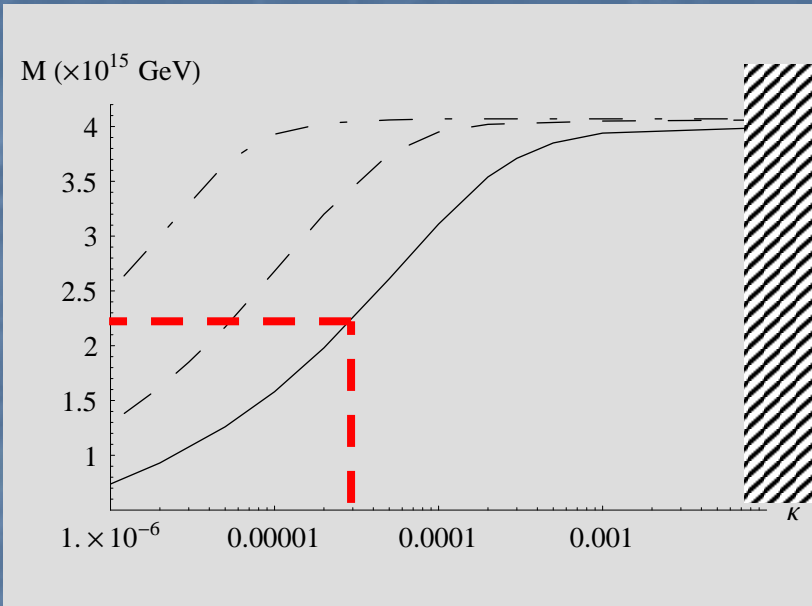
We obtain a single unknown equation in  $x_Q$  for a given  $\kappa$ ,  $N=3$  and  $N_Q=60$  :

$$\left(\frac{\delta T}{T}\right)_{Q\text{-COBE}}^2 = y_Q^{-4} \left(\frac{\kappa^2 N N_Q}{32\pi^2}\right)^2 \left[ \frac{64 N_Q}{45 N} x_Q^{-2} y_Q^{-2} f^{-2}(x_Q^2) + \left(\frac{0.77 \kappa}{\pi}\right)^2 + 324 \right]$$

Thus we obtain  $x_Q(\kappa)$  and thus  $M(\kappa)$ . We can also calculate the strings contribution for a given  $\kappa$  defining

$$A_{CS} = \left(\frac{\delta T}{T}\right)_{Q\text{-strings}}^2 / \left(\frac{\delta T}{T}\right)_{Q\text{-tot}}^2$$

# Constraints on F term inflation



- Gravitino constraint on the reheating temperature impose  $\kappa < 8 \times 10^{-3}$
- In WMAP, the cosmic strings contribution to the CMB anisotropies is lower than 9% (99% CL). [Pogosian et al. astro-ph/0403268]

Consequences :  $\kappa < 3 \times 10^{-5}$  ← fine tuning of the coupling

$$M < 2.2 \times 10^{15} \text{ GeV}$$

# The D term inflation

Introduction of an additional U(1) factor with a gauge coupling  $g$  and a non vanishing FI term  $\xi$ . [ Charges under U(1) :  $Q(\Phi_{\pm})=\pm 1$ ,  $Q(S)=0$  ]

Superpotential :  $W^D = \lambda S \Phi_+ \Phi_-$

In the SUSY framework : the inflaton field reaches the PLANCK mass!!

⇒ **Supergravity** is needed.

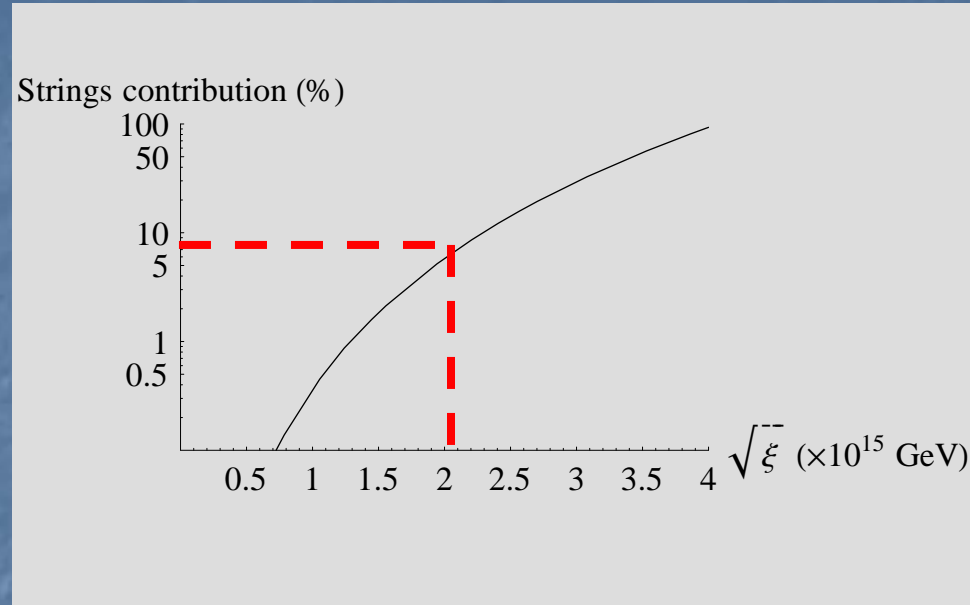
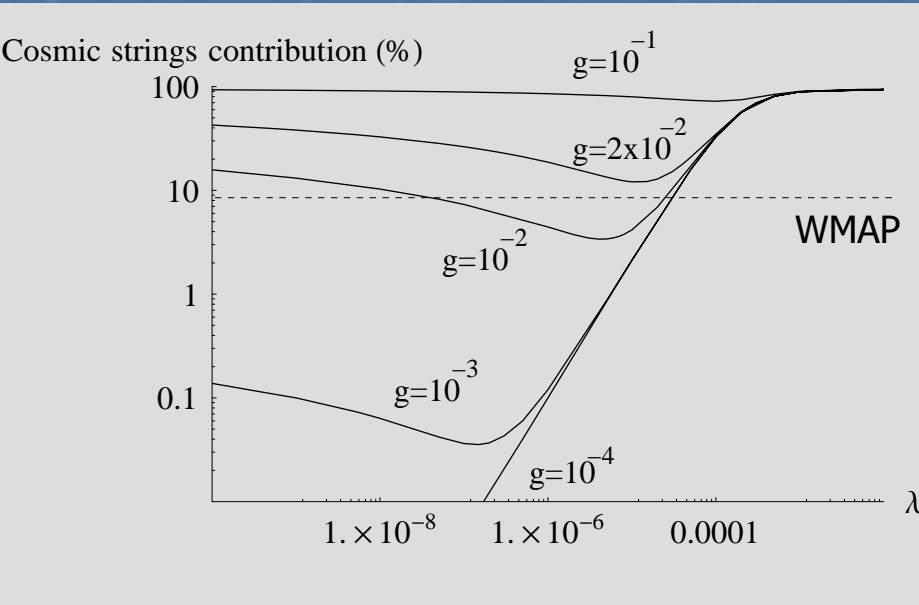
→ Study in the minimal supergravity :

$$K = |S|^2 + |\phi_+|^2 + |\phi_-|^2$$

→ Completely numerical resolution.

We obtain the cosmic strings contribution as a function of the 3 parameters :  $g, \lambda, \sqrt{\xi}$ .

# Constraints on D term inflation



- The WMAP constraint on cosmic string contribution to the CMB imposes

$$g < 2 \times 10^{-2}$$

$$\lambda < 3 \times 10^{-5}$$

$$\sqrt{\xi} < 2 \times 10^{15} \text{ GeV}$$

- The SUGRA corrections induce a lower limit on  $\lambda$ .

# Conclusions

- The **GUT** cosmic strings are **compatible** with current CMB measurements.
- Their low influence on the power spectrum can be used to **constrain** inflationary models
  - For F-term inflation, both the superpotential coupling  $\kappa$  and the mass scale  $M$  can be constrained.
  - For D-term inflation, the gauge  $g$  and the superpotential  $\lambda$  coupling constant as well as the FI term  $\xi$  can be constrained.

The **SUGRA** framework is necessary. This is still an **open** possibility.

- For both, we obtain a mass scale ( $M$  or  $\sqrt{\xi}$ ) smaller than  $2 \times 10^{15}$  GeV and a superpotential coupling ( $\kappa$  or  $\lambda$ ) smaller than  $3 \times 10^{-5}$ .

These models suffer from some **fine tuning**...