Primordial non-Gaussianity

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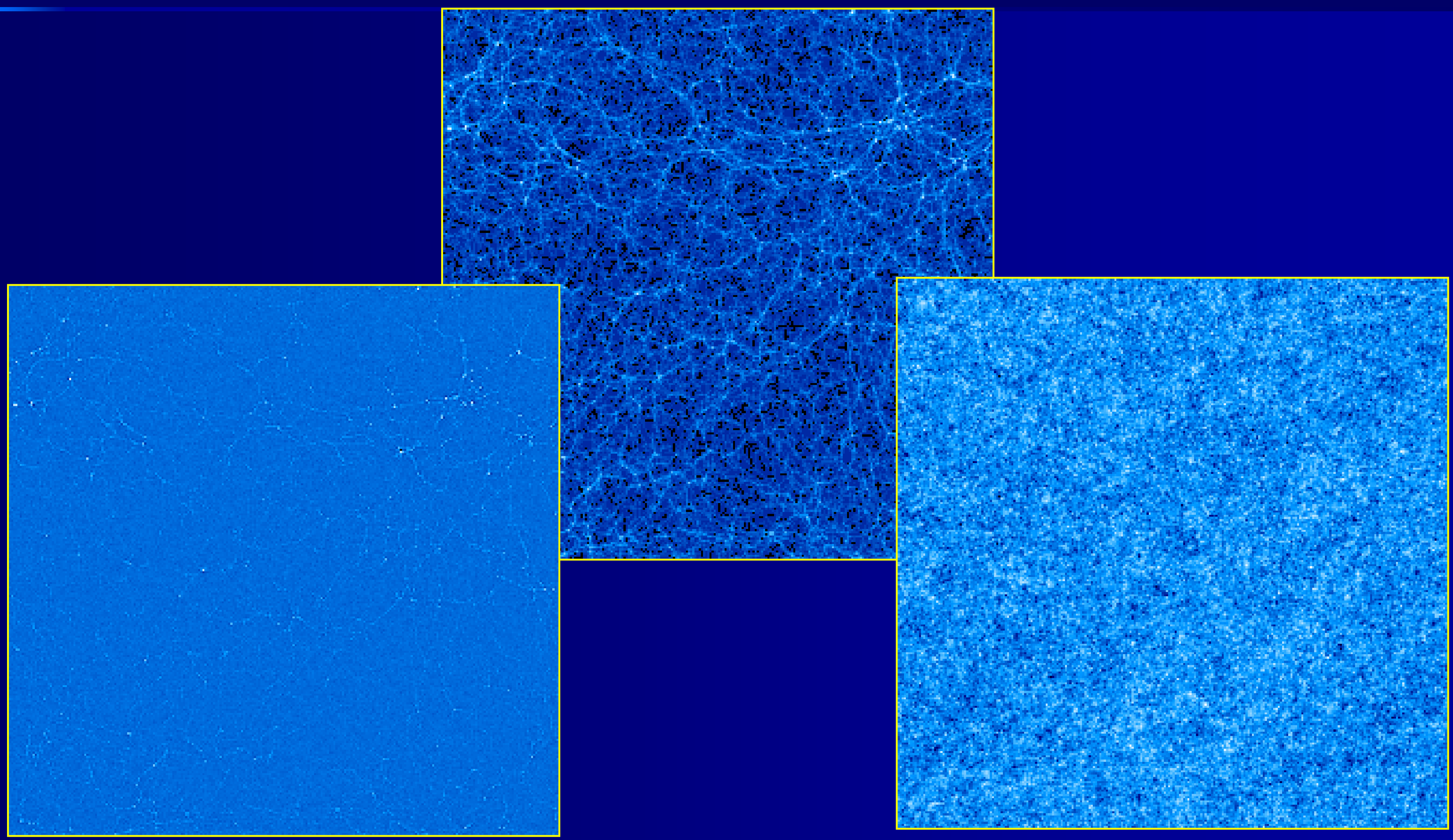
based on ...

- Acquaviva V., Bartolo N., Matarrese S. & Riotto A. 2003, Nucl. Phys. B **667** 119
- Bartolo N., Komatsu E., Matarrese S. & Riotto A. 2004, to appear in Phys. Rept., astro-ph/0406398
- Bartolo N., Matarrese S. & Riotto A. 2002, Phys. Rev. D **65** 103505
- Bartolo N., Matarrese S. & Riotto A. 2004, Phys. Rev. D **69** 043503
- Bartolo N., Matarrese S. & Riotto A. 2004, JCAP **01** 003
- Bartolo N., Matarrese S. & Riotto A. 2004, JHEP **04** 006
- Liguori M., Matarrese S. & Moscardini L. 2003, Ap. J. **597** 56
- Matarrese S., Musso, M.A., Riotto, A., 2004, JCAP, **05** 008
- Mollerach S. & Matarrese S., 1997, Phys. Rev. D **56** 4494

review on
non-Gaussianity
from Inflation

The effect of phases



credits: *Peter Coles*

The non-Gaussian model

✓ Many primordial (inflationary) models of non-Gaussianity can be represented in configuration space by the general formula (e.g. *Verde et al. 2000; Komatsu & Spergel 2001*)

$$\Phi = \phi_L + f_{NL} * (\phi_L^2 - \langle \phi_L^2 \rangle)$$

where Φ is the large-scale gravitational potential, ϕ_L its linear Gaussian contribution and f_{NL} is the dimensionless *non-linearity parameter* (or more generally *non-linearity function*). The percent of non-Gaussianity in CMB data implied by this model is

$$NG\ \% \sim 10^{-5}\ |f_{NL}|$$

$< 10^{-3}$ from *WMAP*

Classify Inflationary Models (I)

- ✓ The shape of the inflaton potential $V(\varphi)$ determines the observables.

slow-roll conditions

- ✓ It is standard to use three parameters to characterize the shape:

ε "slope" of the potential $\sim (V'/V)^2$ $\ll 1$

η "curvature" of the potential $\sim V''/V$ $\ll 1$

ξ "jerk" of the potential $\sim (V'/V)(V'''/V) \sim \varepsilon^2$

Classify Inflationary Models (II)

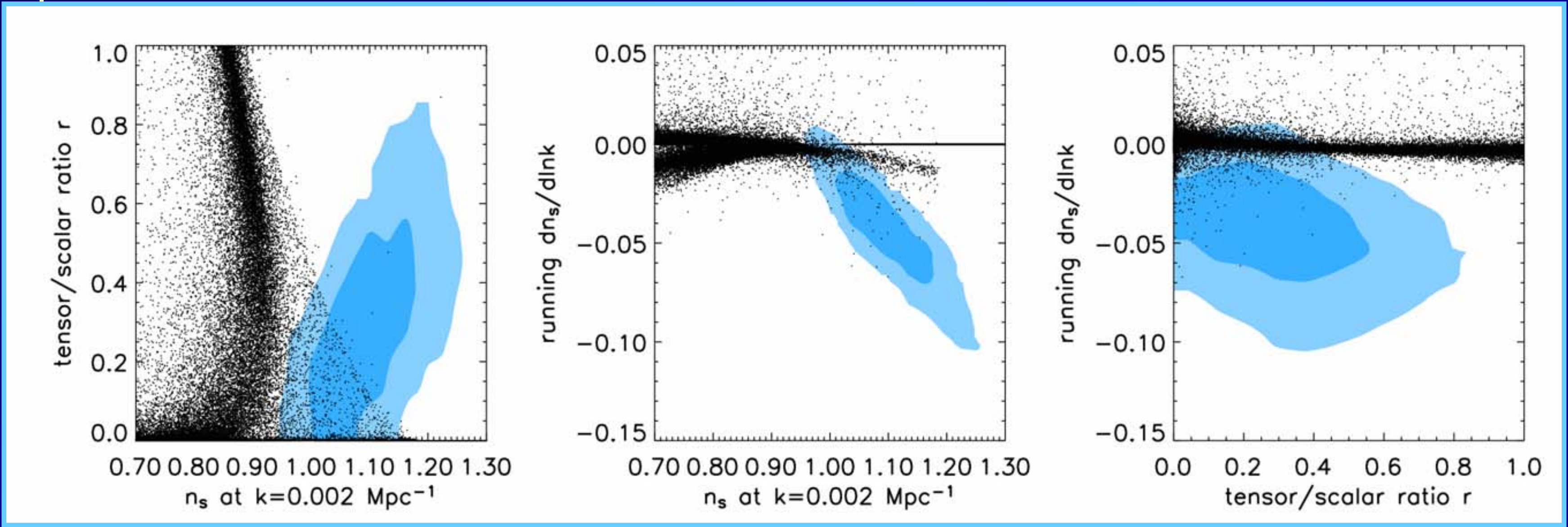
Ratio of tensor to scalar modes

Tilt of scalar perturbations

Running scalar spectral index

$$\begin{cases} r = 16\varepsilon \\ n_s = 1 - 6\varepsilon + 2\eta \\ dn_s / d\ln k = -2\xi + 16\varepsilon\eta - 24\varepsilon^2 \end{cases}$$

"Generic" predictions of single field slow-roll models vs. WMAP



Peiris et al. 2003

Each point is a "viable" slow-roll model, able to sustain inflation for sufficient *e*-foldings to solve the horizon problem and make the Universe (nearly) flat.

Monte Carlo simulations following (*Kinney 2002*) flow-equation technique

Where does large-scale non-Gaussianity come from?

- *Falk et al. (1993)* found $f_{NL} \sim \xi \sim \varepsilon^2$ (from non-linearity in the inflaton potential in a fixed de Sitter space) in the standard single-field slow-roll scenario
- *Gangui et al. (1994)*, using stochastic inflation found $f_{NL} \sim \varepsilon$ (from second-order gravitational corrections during inflation). *Acquaviva et al. (2003)* and *Maldacena (2003)* confirmed this estimate (up to numerical factors and momentum-dependent terms) with a full second-order approach
- *Bartolo et al. (2004)* show that second-order corrections <u>after inflation</u> enhance the primordial signal leading to $f_{NL} \sim 1$

Non-Gaussianity requires more than linear theory ...

The leading contribution to higher-order statistics (such as the bispectrum, i.e. the FT of the three-point function) comes from second-order metric perturbations around the RW background (unless the primordial non-Gaussianity is very strong)

"*... the linear perturbations are so surprisingly simple that a perturbation analysis accurate to second order may be feasible ...*" (Sachs & Wolfe 1967)

<u>First-order</u> metric perturbations in the Newtonian gauge (*dust case*)

$$ds^2 = a^2(\tau)[-(1+2\phi)d\tau^2 - 2V_i d\tau dx^i + ((1-2\psi)\delta_{ij} + h_{ij})dx^i dx^j]$$

$$\nabla^2\nabla^2(\phi-\psi) = 0$$ ← scalar modes

$$\nabla^2\nabla^2 V_i = 0$$ ← vector modes

$$V^i_{,i} = 0$$

tensor modes →

$$\nabla^2\nabla^2(\ddot{h}_{ij} + H\dot{h}_{ij} - \nabla^2 h_{ij}) = 0$$

$$h_{ij} = h_{ji} \qquad h^i_i = h^i_{j,i} = 0$$

Second-order metric perturbations in the Poisson gauge (*dust case*)

$$ds^2 = a^2(\tau)[-(1+2\phi)d\tau^2 - 2V_i d\tau dx^i + ((1-2\psi)\delta_{ij} + h_{ij})dx^i dx^j]$$

$$\nabla^2\nabla^2(\phi - \psi) = -2\nabla^2\nabla^2\varphi^2 - \frac{1}{2}\nabla^2(2\partial^l\varphi\partial_l\varphi + 3H^2v^2)$$

$$+\frac{3}{2}\partial^i\partial_j(2\partial_i\varphi\partial^j\varphi + 3H^2 v_i v^j)$$

scalar modes

$$\nabla^2\nabla^2 V_i = 16\pi G a^2\partial^j(v_j\partial_i\rho - v_i\partial_j\rho)$$

vector modes

$$\nabla^2\nabla^2(\ddot{h}_{ij} + H\dot{h}_{ij} - \nabla^2 h_{ij}) = 2[\nabla^2\partial^k\partial_l R^l_k\delta^i_j$$

$$+2\nabla^2(\nabla^2 R^i_j - \partial^k\partial_j R^i_k - \partial^i\partial_l R^l_j)$$

$$+\partial^i\partial_j\partial^k\partial_l R^l_k]$$

tensor modes

$$R^i_j \equiv \partial^i\varphi\partial_j\varphi - \frac{1}{2}(\nabla\varphi)^2\delta^i_j + 4\pi G a^2\rho(v^i v_j - \frac{1}{3}v^2\delta^i_j)$$

Extended to fully non-linear scales by Carbone & Matarrese (2004)

Non-Gaussianity from Inflation: results (I)

- The amount of non-Gaussianity from a wide class of models, including single-field slow-roll inflation, curvaton (*Mollerach 1990; Moroi & Takahashi 2001; Enqvist & Sloth 2002; Lyth & Wands 2002*) and modulated reheating (*Hamazaki & Kodama 1996; Dvali et al. 2003; Zaldarriaga 2003; Kofman 2003*), once second-order effects are accounted for, follows a universal relation.

- One should also account for i) the large-scale (Sachs-Wolfe-like) second-order temperature fluctuations (*Mollerach & Matarrese 1997*), ii) the second-order intrinsic temperature anisotropies at last-scattering, and for iii) the effect of angular deflection in angular averaged relations, i.e. the first-order metric determinant (*Bartolo et al. 2004*)

Evaluating non-Gaussianity from inflation

- Evaluate non-Gaussianity during inflation by a self-consistent second-order calculation.
- Evolve scalar (vector and tensor) perturbations to second order after inflation outside the horizon, matching a conserved second-order gauge-invariant variable, such as the comoving curvature perturbation $\zeta^{(2)}$ defined by *Malik & Wands (2004*), or the similar quantity defined by *Salopek & Bond (1990)*, $\zeta^{(2)}_{SB} = \zeta^{(2)}|_\rho - 2\,(\zeta^{(1)})^2$, to its value at the end of inflation (accurately accounting for the Universe reheating after inflation).
- Evolve them consistently inside the horizon → this should involve a calculation of the radiation transfer function to second order!

Large-scale second-order CMB anisotropies

$$\Delta T/T = (\phi + \tau)_E - \phi_E^2/2 + \phi_E \tau_E + \text{integrated effects}$$

(*Pyne & Carroll 1996, Mollerach & Matarrese 1997*)

expand ϕ and τ to second order

$$\phi = \phi^{(1)} + \phi^{(2)}/2, \qquad \tau = \tau^{(1)} + \tau^{(2)}/2$$

$$\Delta T/T = (1/3)[\phi^{(1)}{}_E + \phi^{(2)}{}_E/2 - 5(\phi^{(1)}{}_E)^2/6]$$

and estimate $\phi^{(2)}$ from inflation and post-inflation dynamics

Non-Gaussianity from Inflation: results (II)

$\Delta T/T = \mathbf{1/3}\,(\phi_L + \phi_{NL})$

Sachs-Wolfe limit; replaced by full transfer function in CMB maps

$\phi_{NL} = f_{NL} * \phi_L^2 + const.$

$f_{NL} = f_{NL}^0 - 3(k_1^4 + k_2^4)/2k^4 + (\underline{k}_1 \cdot \underline{k}_2 / k^2) \cdot$
$\cdot[4 - 3(\underline{k}_1 \cdot \underline{k}_2 / k^2)], \qquad \underline{k} = \underline{k}_1 + \underline{k}_2$

Non-linearity parameter

- *Standard single-field slow-roll inflation:*

  $f_{NL}^{0} = 7/3$

- *Curvaton scenario:*

  $f_{NL}^{0} = 2/3 - 5r/6 + 5/4r$

  r = ratio of curvaton to radiation energy-densities at curvaton decay

- *Modulated (inhomogeneous) reheating scenario:*

  $f_{NL}^{0} = 13/12 - I$

  I = 0 for the minimal case

Inflation models yielding $f_{NL} \sim 100$ (adiabatic mode)

- ✓ **Multi-field inflation** (*Bartolo et al. 2002; Bernardeau & Uzan 2002; Bernardeau et al. 2004; Enqvist & Vaihkonen 2004; Rigopoulos & Shellard 2004*) giving rise to a cross-correlated and generally non-Gaussian mixture of isocurvature and adiabatic perturbations (constrained by *WMAP* data; *Komatsu, Spergel & Wandelt 2003*)

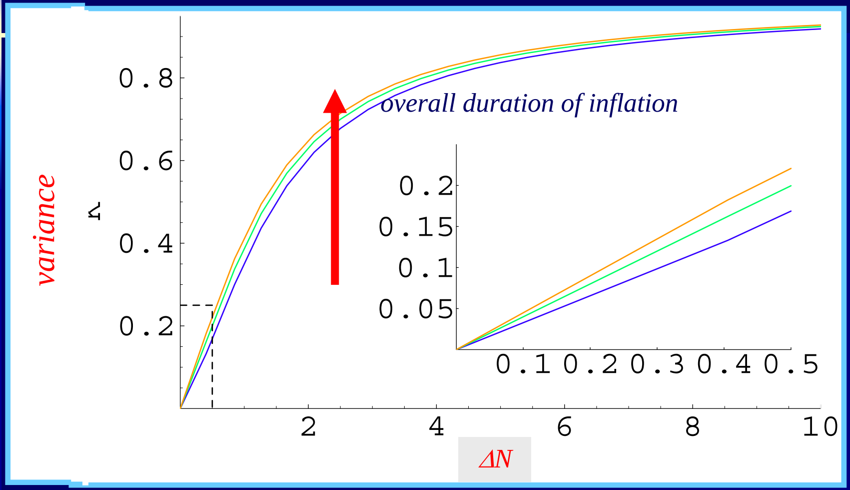
unconventional models:

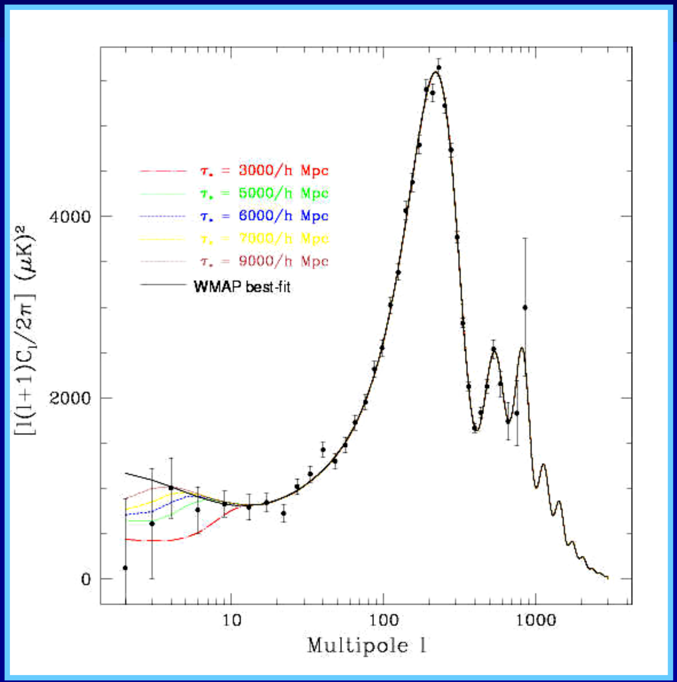
- ✓ **Higher-order operators** (*Creminelli 2003*)
- ✓ **Ghost inflation**: inflation driven by a ghost condensate (*Arkani-Hamed et al. 2004*)
- ✓ **D-cceleration**: strong coupling QFT effects sum to provide a Dirac-Born-Infeld action for the inflaton (*Alishahiha et al. 2004; Silverstein & Tong 2004*)

Inflation models and f_{NL}

model	$f_{NL}(\mathbf{k}_1,\mathbf{k}_2)$	comments
single-field inflation	$7/3 - g(\mathbf{k}_1, \mathbf{k}_2)$	$g(k_1, k_2)=3(k_1^4+k_2^4)/2k^4+(\underline{k}_1 \cdot \underline{k}_2) \cdot [4-3(\underline{k}_1 \cdot \underline{k}_2)/k^2]/k^2$, $\underline{k}=\underline{k}_1+\underline{k}_2$
curvaton scenario	$2/3 - 5r/6 + 5/4r - g(\mathbf{k}_1, \mathbf{k}_2)$	$r \sim (\rho_\sigma/\rho)_{decay}$
modulated reheating	$13/12 - I - g(\mathbf{k}_1, \mathbf{k}_2)$	$I = - 5/2 + 5\Gamma / (12\, \alpha\Gamma_1)$ $I = 0$ (*minimal case*)
multi-field inflation	up to 10^2	*order of magnitude estimate of the absolute value*
"unconventional" inflation set-ups		
warm inflation	typically 10^{-1}	*second-order corrections not included*
ghost inflation	$- 140\, \beta\, \alpha^{-3/5}$	*post-inflation corrections not included*
D-cceleration	$- 0.1\, \gamma^2$	*post-inflation corrections not included*

Influence of super-horizon scales on sub-horizon observables





- ✓ Further scale-dependent non-Gaussianity is produced by the cross-talk of super-horizon and sub-horizon scales, provided one accounts for the coloured (*non-Markovian*) nature of the sub-horizon quantum noise.

- ✓ Using the techniques of out-of-equilibrium field theory one finds memory effects which produce: a ***blue tilt*** (*Liguori, Matarrese, Musso & Riotto 2004*) of the power-spectrum on the largest scales and an ***excess of non-Gaussianity*** also on large scales, whose size depends on the overall duration of inflation (*Matarrese, Musso & Riotto 2004*).

Non-Gaussian CMB anisotropies: map making

Liguori, Matarrese & Moscardini 2003, ApJ **597**, 56

- assume mildly non-Gaussian large-scale potential fluctuations

$$\Phi(\underline{\mathbf{r}}) = \Phi_L(\underline{\mathbf{r}}) + f_{NL}\Phi_L^2(\underline{\mathbf{r}})$$

- account for radiative transfer

$$a_{\lambda m} = \frac{2}{\pi}\int dr\ r^2 \Delta_\lambda(r) \int d\Omega_{\hat{r}} \Phi(\underline{\mathbf{r}}) Y^*_{\lambda m}(\hat{\mathbf{r}})$$

radiation transfer functions

harmonic transform: $\Phi_{lm}(\underline{\mathbf{r}})$

Spherical coordinates in real space (I)

Work directly with multipoles in real space (to avoid Bessel transform and Cartesian coordinates)

1. generate white noise coefficients $n_{lm}(r)$
2. cross-correlate different $n_{lm}(r)$ by a convolution with suitable filters $W_l(r,r_1)$

$$\Phi_{\lambda m}(r) = \int dr_1 r_1^2 \; W_\lambda(r, r_1) \; n_{\lambda m}(r_1)$$

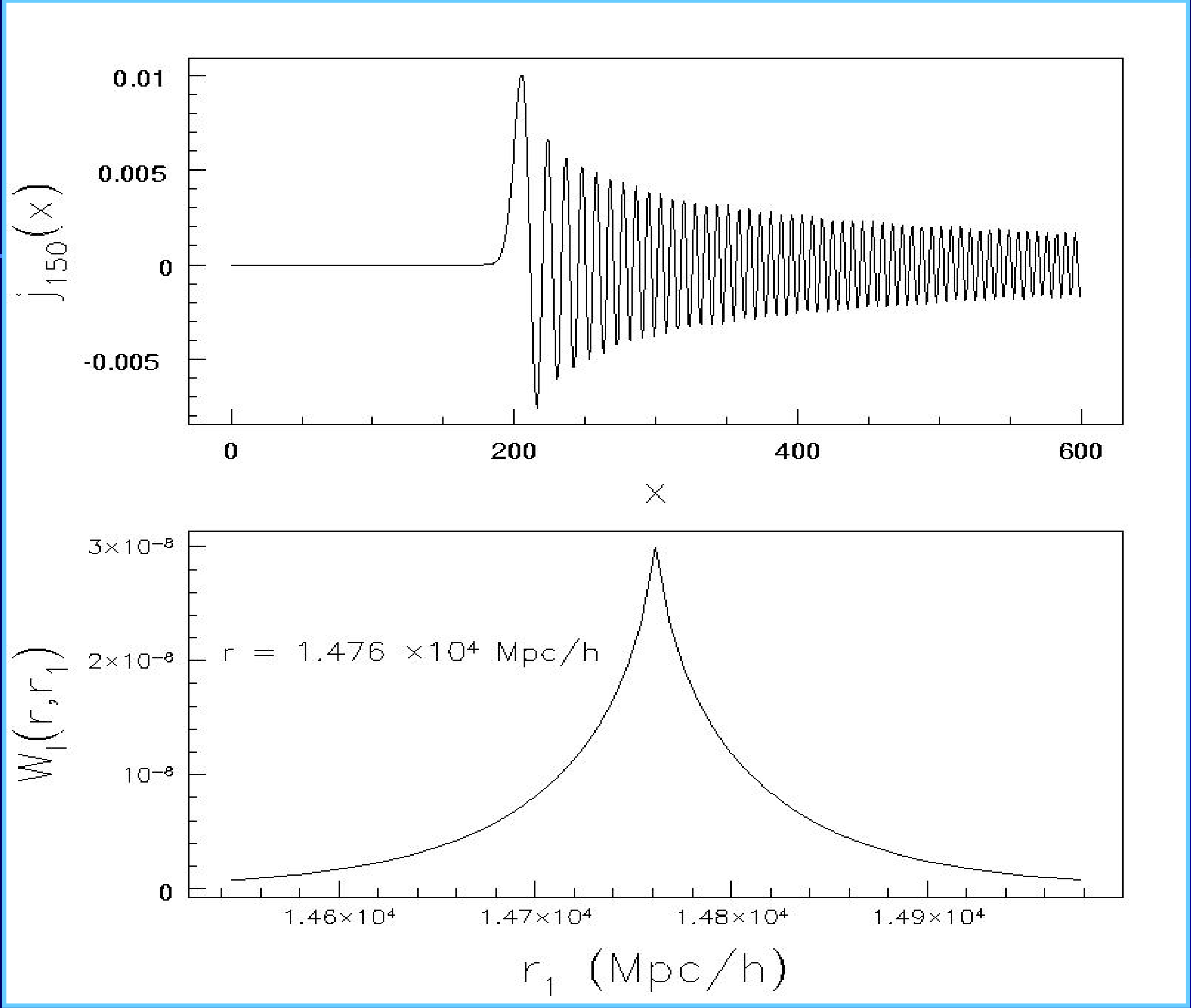
Spherical coordinates in real space (II)

band-pass filters $W_l(r,r_1)$

$$W_\lambda(r,r_1)=\int dk\,k^2\sqrt{P_\Phi(k)}\,j_\lambda(kr)\,j_\lambda(kr_1)$$

linear gravitational potential power-spectrum

$W_l(r,r_1)$ does not oscillate as fast as $j_l(kr)$


0.01
0.005
0
-0.005
$j_{150}(x)$
0
200
400
600
x
3×10^{-8}
2×10^{-8}
10^{-8}
0
$W_l(r,r_1)$
$r = 1.476 \times10^4$ Mpc/h
1.46×10^4
1.47×10^4
1.48×10^4
1.49×10^4
r_1 (Mpc/h)

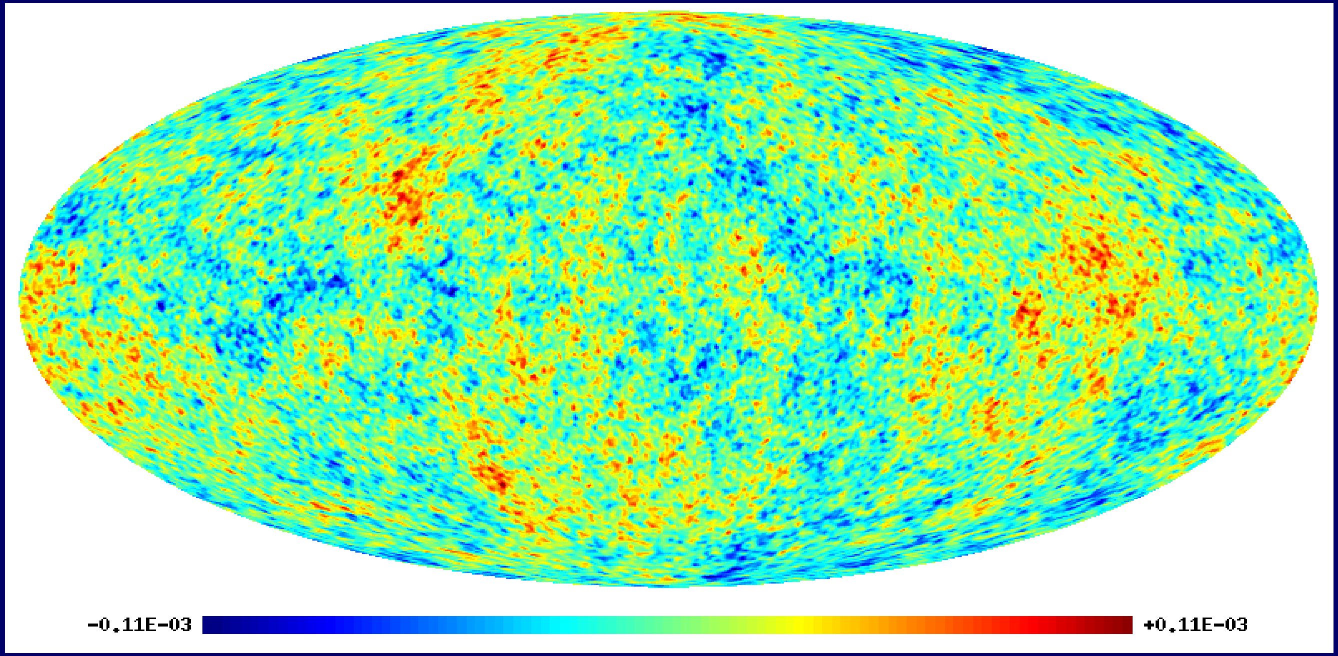
Outline of the code

1. precompute transfer functions (extracted from CMBfast) for a given model
2. precompute filters $W_l(r,r_1)$
3. generate white-noise coefficients $n_{lm}(r)$
4. correlate white-noise coefficients to find multipoles $\Phi^{L}_{lm}(r)$ and then extract $\Phi^{NL}_{lm}(r)$
5. obtain CMB multipoles by convolving with radiation transfer function

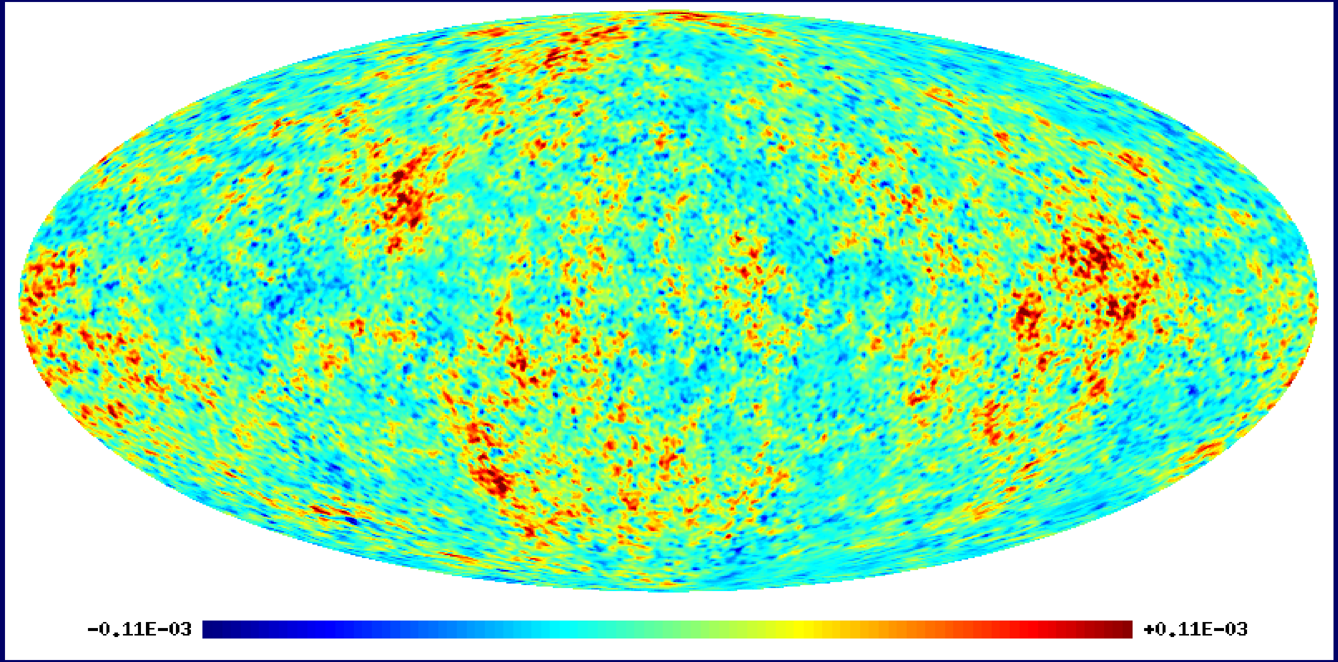
CPU time for a single map

l_{max}	CPU time (hh:mm) *	harmonic transform (hh:mm)	N_{side}
300	00:08	00:06	128
500	00:40	00:32	256
750	01:08	01:00	256
1500	17:34	16:00	512
3000	137:30	134:30	1024

* parallelization of the code is in progress



-0.11E-03
+0.11E-03



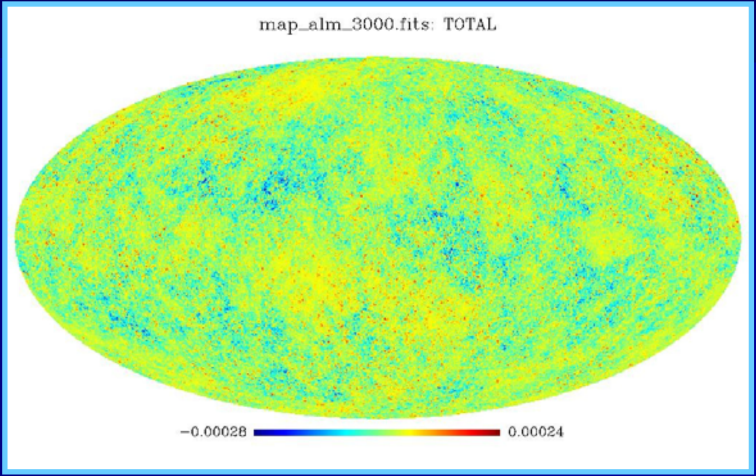
-0.11E-03
+0.11E-03

Non-Gaussian CMB maps: Planck resolution

5’ resolution

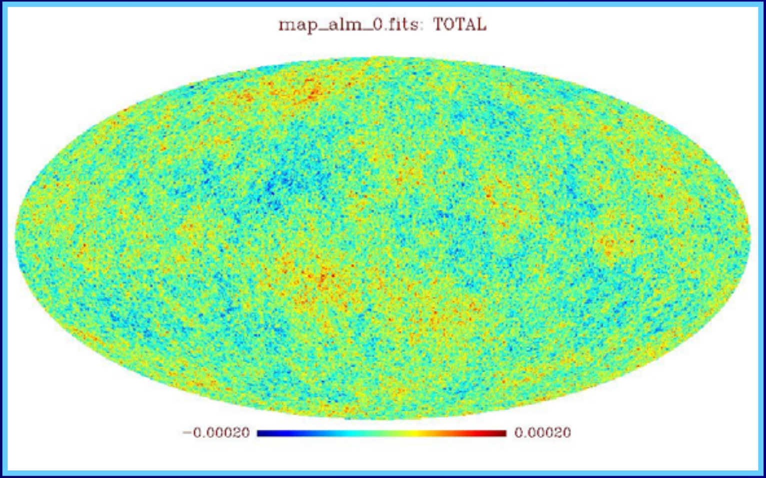
l_{max} = 3000, N_{side}=2048

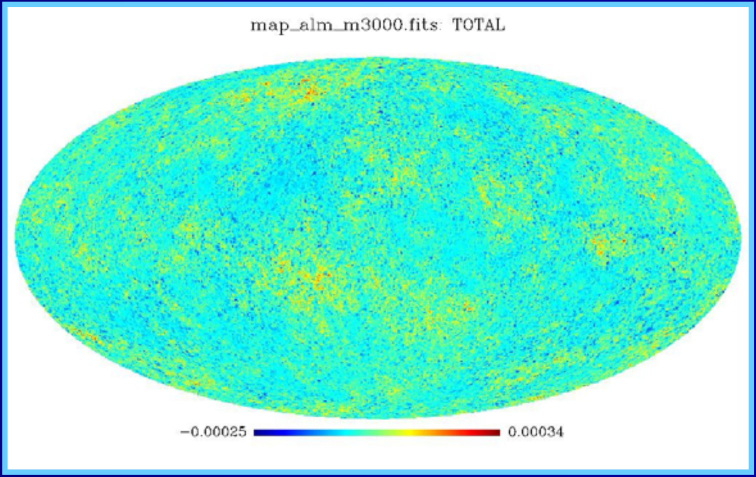
f_{NL} = 3000



Liguori, Matarrese & Moscardini 2003, ApJ **597**, 56

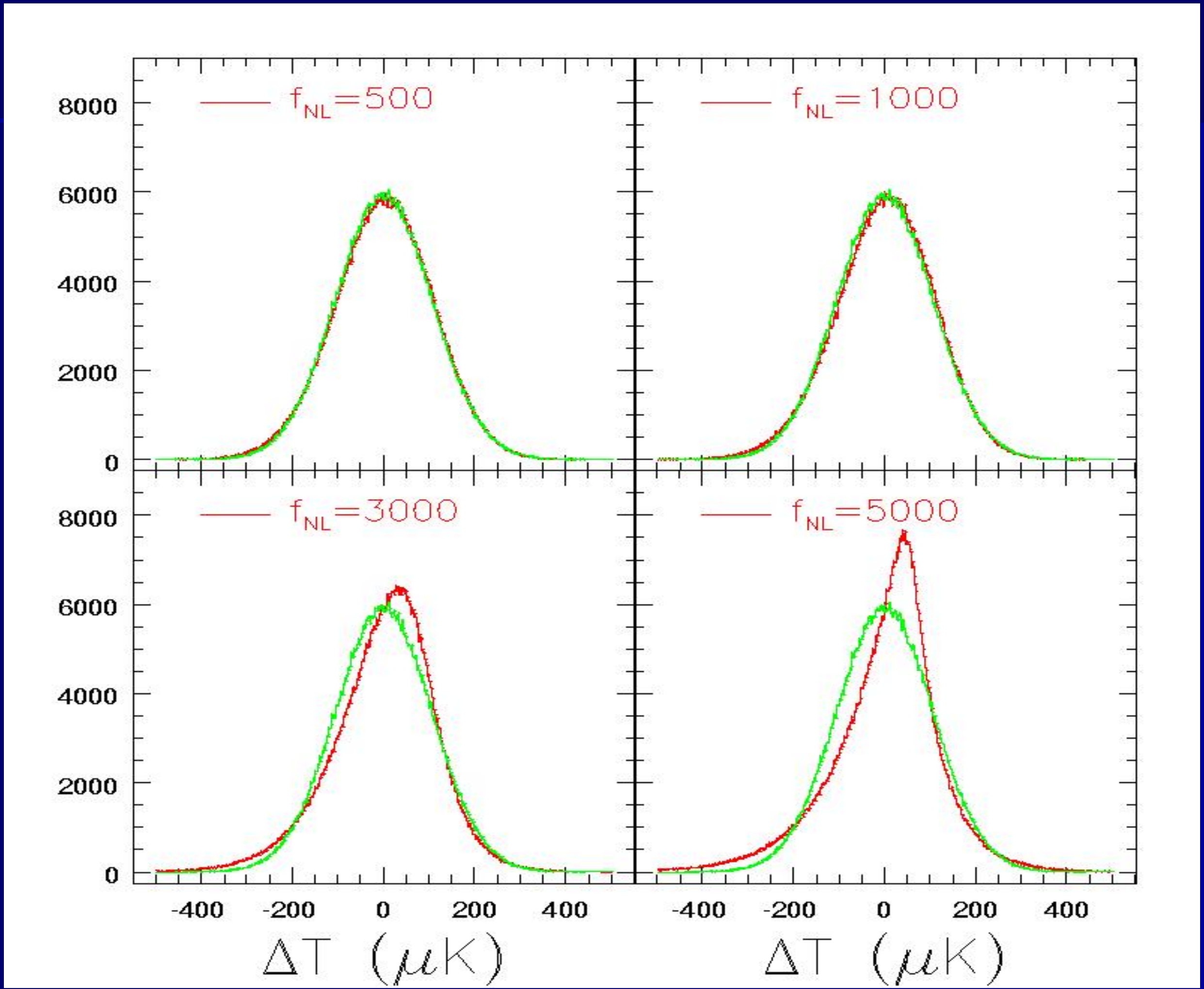
f_{NL} = 0





f_{NL} = - 3000

PDF of the NG CMB maps



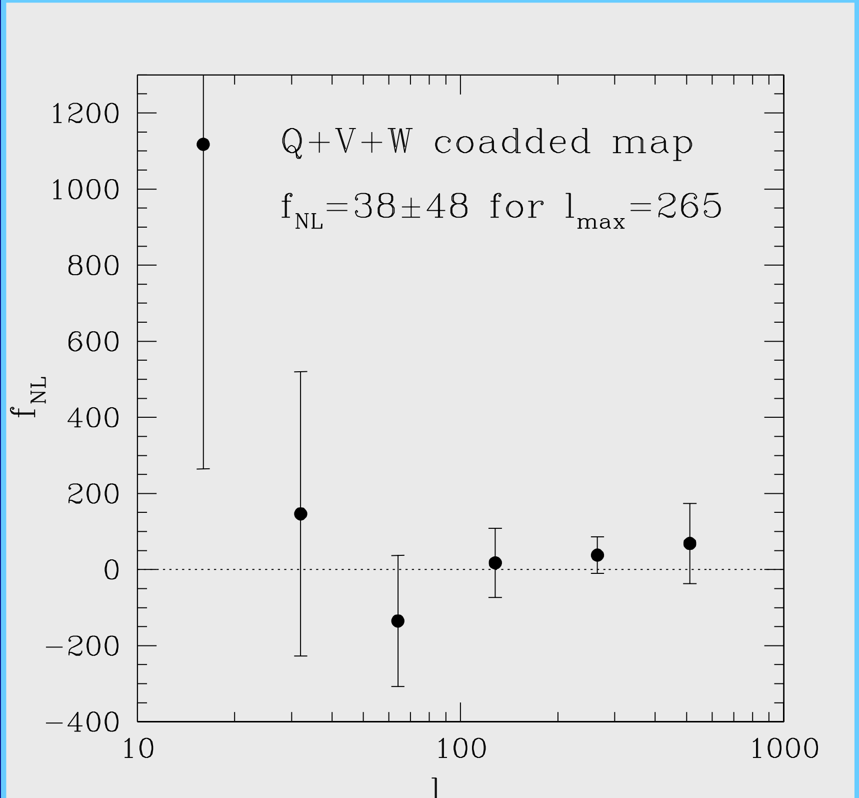
Observational constraints on f_{NL}

- The strongest limits on non-Gaussianity so far come from WMAP data. *Komatsu et al. (2003)* find (at 95% cl)

  $-58 < f_{NL} < 134$

- According to *Komatsu & Spergel (2001)* using the angular bispectrum one can reach values as low as $|f_{NL}| = 20$ with *WMAP* & $|f_{NL}| = 5$ with *Planck* can be achieved

- The role of the f_{NL} momentum-dependent part is being explored (*Liguori, Matarrese & Riotto 2004*) as a characteristic inflation signature refcelting in some specific triangle configuration of the bispectrum



Komatsu *et al.* 2003

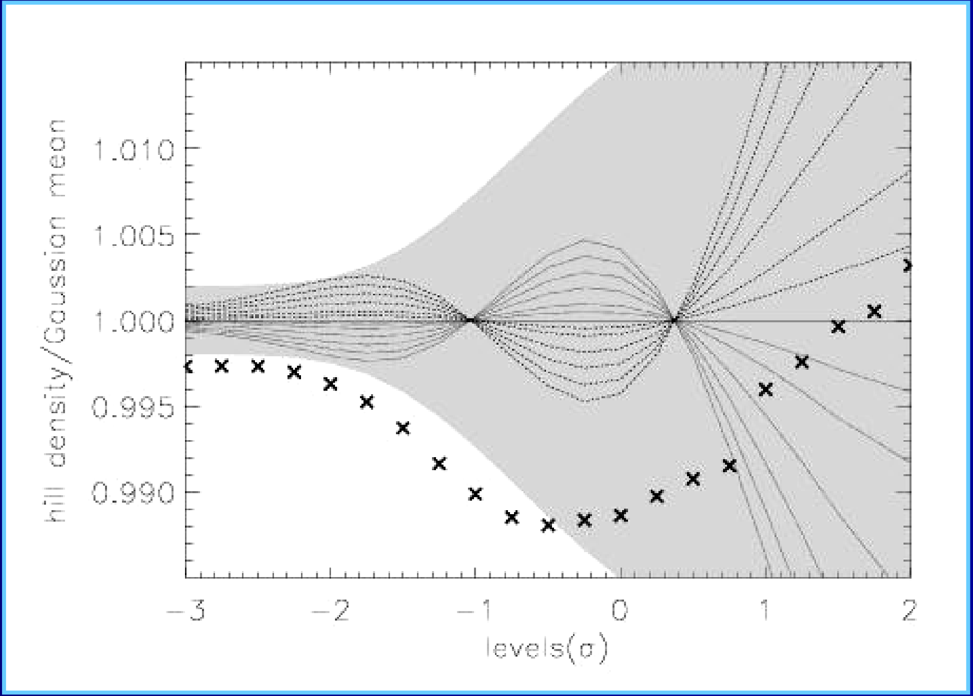
Statistical analysis of NG CMB maps vs. WMAP

Local curvature of CMB anisotropies

Density of hills (where the Hessian eigenvalues are both positive) as a function of the threshold ν, for different values of f_{NL}. The grey band is the 1σ confidence level. The solid crosses are the *WMAP* data. One finds

$$f_{NL} = 30 \pm 210$$

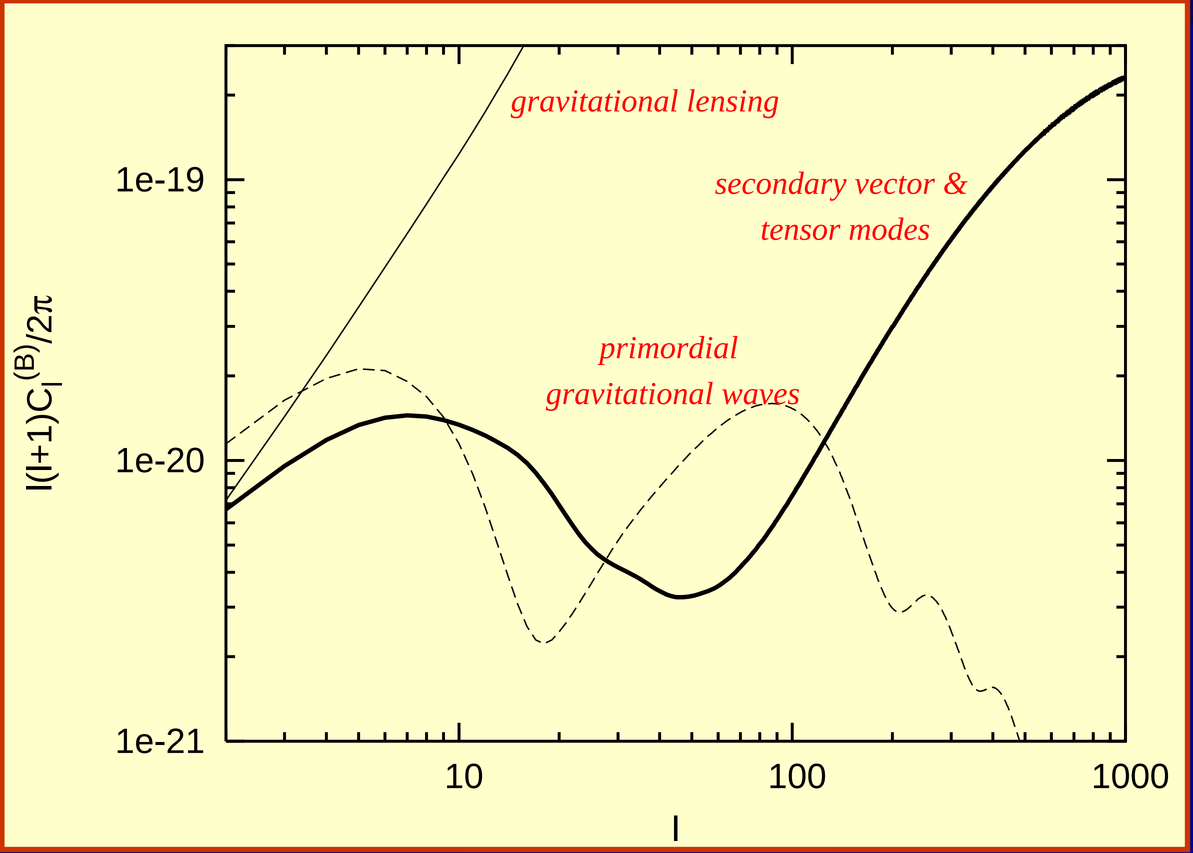
at the 2σ level



Cabella P., Liguori M., Hansen F., Marinucci D., Matarrese S., Moscardini L. & Vittorio N. 2004

Second-order effects from scalar modes & B-mode polarization

The B-mode polarization produced by primordial gravitational waves can be hidden by gravitational lensing and/or by second--order vector and tensor modes, unless the inflation energy scale is larger than 10^{15} GeV



Mollerach, Harari & Matarrese 2004, Phys. Rev. D ***69*** *063002*

Conclusions & future prospects

- ✓ Contrary to earlier naive expectations, some level of non-Gaussianity is generically present in all inflation models
- ✓ The level of non-Gaussianity predicted in the simplest inflation models is slightly below the minimum value detectable by *Planck*, but the predicted angular dependence of f_{NL}, extensive use of simulated non-Gaussian CMB maps, measurements of polarization and use of alternative statistical estimators might help non-Gaussianity detection down to $f_{NL} \sim 1$
- ✓ Constraining or detecting non-Gaussianity will become a powerful tool to discriminate among competing scenarios for perturbation generation (*standard slow-roll inflation, curvaton and modulated-reheating scenarios, multi-field or ghost inflation, …*) some of which imply large non-Gaussianity
- ✓ Accounting for the presence of sizeable non-Gaussianity in maximum likelihood analyses might change the estimated value of cosmological parameters
- ✓ Predicting or constraining non-Gaussianity should be considered as a branch of *Precision Cosmology*