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COMBINING INTERFEROMETRIC SUNYAEV-ZEL'DOVICH EFFECT MEASUREMENTS AND WEAK LENSING

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Weak lensing and the Sunyaev-Zel'dovich effect (SZE) are each measures of the projected mass distribution of galaxy clusters. Weak lensing is probing the projected total mass, while the SZE is probing the projected (temperature-weighted) gas distribution. Direct comparisons of the two distributions should allow determinations of the gas mass fraction and the gas temperature. With a local calibration of the true gas fraction, it should be possible to estimate angular diameter distances to clusters as well. Interferometric measurements must be handled carefully, however, with the proper analysis done in the Fourier domain, rather than on the reconstructed images. We present a non-parametric method for comparing the SZE and mass distributions that properly accounts for the finite fields of view of both the interferometer and the weak lensing shear measurements. At the sensitivity limits of current instruments, we estimate that the gas fraction can be determined to an accuracy better than 20% if the gas temperature is known from other methods, while the temperature can be estimated to a comparable precision if the redshift distribution of lensed sources is approximately known.

1 Introduction

Both the Sunyaev-Zel'dovich effect (SZE) and weak lensing measurements are measures of the projected mass distribution of galaxy clusters. SZE maps give a measure of the temperature-weighted projected gas mass, while weak lensing is sensitive to the projected total mass. A simple comparison of the two maps could yield valuable information on both the gas mass fraction and temperature structure in the ICM.

In practice, however, the observing strategy will always play a significant role in the data interpretation, and in this work we will address the comparison of interferometric SZE measurements and weak lensing shear measurements within a finite field. Interferometric measurements of the SZE and weak lensing measurements share the feature that they are both well-suited to an analysis in the Fourier domain. The output of an interferometer is directly proportional to the Fourier transform of the sky distribution multiplied by a "primary beam," making interferometric measurements most easily understood in the Fourier domain. The measured shear (for weak lensing) is proportional to gradients of the projected potential, which is in turn connected to the projected mass through the two-dimensional analogue of Poisson's equation. These gradient operators are easily implemented in the Fourier domain, allowing the Fourier transform of the shear to be simply related to the Fourier transform of the mass distribution.

In this work, we make these connections more concrete and address the expected accuracy of comparisons between SZE and weak lensing data from current instruments. We will find that comparisons between the two types of data can yield good estimates of the gas fraction and temperature of the ICM, as well as enabling an independent estimate of the angular diameter distance, using only the SZE and weak lensing data.

1.1 *Sunyaev-Zel'dovich Effect*

The Sunyaev-Zel'dovich effect (SZE) [1]. has now been measured for dozens of clusters, with high signal-to-noise maps [2,3]. The thermal SZE is simply a spectral distortion of the CMB by hot electrons along the line of sight. The relatively cool CMB photons are preferentially upscattered by the hot electrons, leading to an intensity decrement at low frequencies and an increment above ~218 GHz

A typical energy exchange per scattering is on the order of kT_e/m_ec^2, the electron temperature in units of the electron rest mass energy, so it is easy to see that the temperature decrement (or increment) will be given by an expression of the form

$$\frac{\Delta T_{SZ}}{T_{\rm CMB}} = g(\nu) \int \frac{kT_e}{m_e c^2} n_e \sigma_T dl \qquad , \tag{1}$$

where n_e is the electron number density and $g(\nu)$ is a dimensionless frequency factor which asymptotes to -2 in the Rayleigh-Jeans limit, has a null near 217 GHz and is positive at larger frequencies. If we assume that the cluster gas is isothermal, or can be characterized by an average temperature, we get a suggestive expression for the temperature

$$\Delta T_{SZ} \propto \Sigma_g \, \overline{T}_e \qquad . \tag{2}$$

For real observations, the observing strategy must be taken into account. An interferometer measures the "visibility", which is the Fourier transform of the sky brightness distribution multiplied by the primary beam $A(\vec{\theta})$;

$$V_{SZ}(u,v) = \frac{\partial B_\nu}{\partial T} \int \Delta T_{SZ}(\vec{\theta}) A(\vec{\theta}) e^{2\pi i \vec{u}\cdot\vec{\theta}} d^2\vec{\theta} \qquad , \tag{3}$$

where B_ν is the familiar Planck function of the blackbody curve and $\vec{u} = (u, v)$ are the conjugate variables to Right Ascension and Declination. Under the assumption of isothermality, we can write the visibility as

$$V_{SZ}(u,v) \propto \overline{T}_e \int \Sigma_g(\vec{\theta}) A(\vec{\theta}) e^{2\pi i \vec{u}\cdot\vec{\theta}} d^2\vec{\theta} \qquad . \tag{4}$$

This multiplication in the image plane can also be written as a simple convolution in the Fourier domain:

$$V_{SZ}(u,v) \propto \overline{T}_e \, \tilde{\Sigma}_g(u,v) \otimes \tilde{A}(u,v) \qquad , \tag{5}$$

using a notation where $\tilde{A}$ denotes the Fourier transform of A and $\otimes$ denotes convolution.

This must be inverted by some non-linear means such as CLEAN or a maximum entropy reconstruction to get an estimate of the sky brightness. This inversion will always be non-unique, since there will always be some Fourier modes which are not measured. For example, there always exist a minimum spacing that corresponds to the diameter of the telescopes. This sets an upper limit to the largest angular scale which can be probed. This non-uniqueness is known as the problem of "invisible distributions"; there are an infinite number of solutions which can take arbitrary values at unmeasured points in the Fourier plane. A map can be reconstructed by making assumptions about the true brightness distribution, but this makes the resulting map subject to biases.

For quantitative studies, it is often better to work in the Fourier plane, with the visibility data. In this plane, the noise is easy to understand and missing spacings are not a problem. Fitting a model of the sky brightness distribution directly to the measured visibilities can be a powerful technique, but care must be taken to ensure that the model is a good approximation to the true distribution. This is a general concern for any parametric technique.

1.2 Weak Lensing

Weak lensing is also becoming a fairly routine method of mapping out clusters at reasonably high signal-to-noise. Weak lensing is the slight distortion of shapes of background galaxies by the gravitational effects of a mass concentration along the line of sight [4]. In this work, we will deal exclusively with weak lensing, where the linearity of the relation between the underlying mass distribution and the measured shear simplifies the problem significantly.

The parameter of interest is the dimensionless surface mass density κ:

$$\kappa \equiv \frac{\Sigma_m}{\Sigma_{crit}} \qquad , \tag{6}$$

The critical surface density for lensing is $\Sigma_{crit} \propto d_A^{-1}\beta^{-1}$, where β encodes the source-lens-observer geometry and therefore has a weak cosmological dependence and d_A is the usual angular diameter distance.

The shear γ is related to this dimensionless surface density, in the weak lensing regime, through the projected potential ϕ:

$$\kappa = \frac{1}{2}(\phi_{11} + \phi_{22}) \quad ; \gamma_1 = \frac{1}{2}(\phi_{11} - \phi_{22}) \ ; \quad \gamma_2 = \phi_{12} \quad , \tag{7}$$

where $\phi_{ij} \equiv \partial^2\phi/\partial x_i \partial x_j$.

A simple way to construct an estimate of κ is to Fourier transform the shear field and take the maximum signal-to-noise combination [4]:

$$\tilde{\kappa}(u,v) = \frac{u^2 - v^2}{u^2 + v^2}\,\tilde{\gamma}_1 \; + \; \frac{2\,u\,v}{u^2 + v^2}\,\tilde{\gamma}_2 \quad . \tag{8}$$

With a measurement of the shear on the whole sky, this relation can be Fourier transformed to reconstruct the projected mass. However, a shear map will have a finite field of view, and it is therefore not possible to construct $\tilde{\gamma}$. Instead, we have $\tilde{\gamma}_i \otimes \tilde{A}_{wl}$, where $\tilde{A}_{wl}$ is the Fourier transform of the field of view of the lensing observation.

Ignoring the effect of the finite field of view can be equivalent to assuming that the shear is zero outside of the field of view. This has the rather unpleasant effect of requiring that the total mass must be zero. In a mass map reconstruction, this usually results in a large negative trough around the central positive mass peak. With a sufficiently large field of view, this negative region can be pushed beyond the region of interest. Alternatively, non-linear methods such as maximum entropy can be used to makes plausible estimates of the cluster mass distribution as a guide to setting the unmeasured shear values to values other than zero.

This situation is very similar to the case of interferometry, where a map reconstruction will be sensitive to the values assigned to unmeasured baselines. While the assumptions need not be very radical to get reasonable and fairly robust maps, a simpler and less model-dependent method would attempt to use the measured data with the least number of implicit assumptions.

2 Making the Interferometric SZE - Finite-Field Weak Lensing Connection

For large radii in the Fourier plane, R_{uv}, the lensing operation in the Fourier plane is slowly varying over the convolving window. This can be seen by looking at equation 8. The operation in the Fourier domain clearly has a typical scale which is proportional to R_{uv}^2. The convolving window, on the other hand, is set by the field of view and will therefore be independent of R_{uv}. Therefore, the variation of the lensing operation over the convolving window becomes less important at large wavenumbers and we can treat it as constant, allowing an estimate of $\tilde{\kappa} \otimes \tilde{A}_{wl}$. This should be compared with the expression for the SZE visibility, equation 5, where we obtained a measure of the gas mass surface density in the Fourier plane convolved with the primary beam.

If the field of view of the shear map is larger than the primary beam of the interferometer, we can simply impose the field of view of the interferometer on the shear map. On the surface, this seems to be throwing away information, but for the most part this is information which is not directly useful for a comparison of the two distributions, since only one data set has measurements of the distribution on these scales. If we take A_{wl} to be the primary beam of the interferometer then we have effectively constructed "weak lensing visibilities" which can be directly compared with the measured SZE visibilities.

At smaller values of R_{uv}, this estimate will be biased slightly low, due to the variation of the lensing operation over the convolving window in the Fourier plane. This can be corrected, if desired, with even a rough model of the cluster, or the resulting visibilities can be simply accepted as lower limits. At large wavenumbers the estimate should be very accurate, but this is also where the signal is much weaker.

Alternatively, with a sufficiently large field of view for the shear map, a direct reconstruction of the mass map can be performed, and the resulting mass map can be multiplied by the primary beam of the interferometer and Fourier transformed. The reconstructed mass map will suffer from the negative trough near the edges, but as long the map is much larger than the interferometer's field of view then this artifact in the map will be strongly weighted down by the primary beam of the interferometer.

The important step is to construct an estimate of a quantity analogous to the SZE visibility in order to compare the gas and mass distributions in the Fourier domain.

3 Gas Fractions, Temperatures and Distances

In comparing the mass and gas distribution, an obvious quantity to study is the gas fraction:

$$f_g = \frac{\Sigma_g}{\Sigma_m} \propto \frac{\Delta T_{SZ}}{\kappa} \frac{\beta \, d_A}{\overline{T}_e} \tag{9}$$

We can construct the same quantity in the Fourier domain, with the projected distributions replaced by their Fourier transforms. As long as we have been careful in constructing our "weak lensing visibilities", we should be able to simply use the SZE visibilities in place of ΔT_{SZ} and the "weak lensing" visibilities in place of κ.

Another estimate of the gas fraction can be constructed by assuming that the gas is in hydrostatic equilibrium and using the SZE data to solve for the underlying mass distribution [5]. The gas mass of the cluster can be directly estimated from its SZE, giving an estimate of the gas fraction:

$$f_g \propto S_{SZ} \, \frac{d_A}{T^2} \quad , \tag{10}$$

where S_{SZ} denotes the integrated SZ flux. Roughly, the gas mass will be proportional to $S_{SZ} d_A^2/T$ and the total mass will be proportional to $d_A T$.

Note that this has the same scaling with angular diameter distance as the gas fraction arising from an SZE-weak lensing comparison. Therefore, a comparison of the two gas fractions will give a distance-independent estimate of the gas temperature:

$$\frac{f_g \ (SZ + WL)}{f_g \ (HSE)} \propto \frac{\beta \, \overline{T}}{\int \kappa \ d^2\vec{\theta}} \tag{11}$$

Although we are comparing the hydrostatic mass (which depends on temperature) to the mass derived from weak lensing, our method does not require that we estimate a total mass from the weak lensing data.

It is quite interesting that we can measure the gas temperature without X-ray data, and that, furthermore, this temperature is independent of the redshift, unlike spectral fitting. Given the gas temperature and a knowledge of the local gas fraction, we could then estimate the angular diameter distance to the cluster in a way that is totally independent of X-ray observations. This offers a remarkable check on distances determined from SZE/X-ray comparisons, and is only limited by the density of lensed sources.

4 Tests and Applications

If the SZE faithfully traces the gas distribution and the gas faithfully traces the mass, then there should be only one value for the gas fraction and it should be the global value. In practice, however, we expect the gas to be in hydrostatic equilibrium with the mass, and therefore rounder and more extended. To what extent the gas fraction will be affected is not clear, but we can use simulations as a guide. We selected three clusters at $z = 0.5$ from an ensemble of clusters simulated in a ΛCDM cosmology and imaged them at an angular diameter distance of 2000 Mpc. The maps are shown in figure 1. We then multiplied the images by a beam of $4.2'$ to simulate the response of OVRO and Fourier transformed the resulting maps.

In figure 2 we show the Fourier profiles of the SZE, gas and mass of three clusters found in n-body hydrodynamical simulations. The top panels show the amplitudes of the Fourier modes and the bottom panels are only the real parts. The Fourier transform was calculated on a 2D grid, and each grid point is shown in this figure. Therefore, the thickness of each curve gives an indication of the degree of azimuthal symmetry in the Fourier transform.

Cluster #2 exhibits significant structure in the Fourier plane, and indeed this cluster is actually a double cluster, as seen in figure 1, with a separation of only roughly $4'$. Overall, the SZE, gas and mass seem to trace each other remarkably well in the Fourier plane. It should be noted, however, that the beam of 4.2' corresponding to OVRO operating at 1 cm will have a minimum spacing in the $u - v$ plane of roughly $1k\lambda$. The primary beam of the interferometer causes some information from shorter spacings to be mixed with the measured visibilities, but this information can only be deconvolved by using multiple pointings.

Even at large spacings, the profiles roughly trace each other. This demonstrates the usefulness of making direct comparisons in the Fourier domain rather than in the image plane. This has the nice property of preserving the Gaussian, uncorrelated noise of the interferometric SZE measurements. The noise in the weak lensing map in the Fourier domain will be highly correlated, but this would be the case for a mass map in the image plane as well.

In order to estimate the practicality of this method, we simulated observations of the simulated clusters. Our method consisted of sampling the visibilities of figure 1 at the positions in the Fourier plane sampled in a real observation of a cluster observed at OVRO and adding appropriate Gaussian noise for the SZE data. We also set up a grid of simulated shear measurements with 20" sampling and added noise to the shear field that was Gaussian distributed with variance 0.1 for the weak lensing data. We repeated this procedure 100 times in order to estimate the uncertainties. The error estimate obtained in this way agreed very well with analytic estimates. For the three clusters, we obtained gas fractions of $0.17 \pm 0.03, 0.13 \pm 0.03$ and 0.18 ± 0.03, using the measured X-ray emission-weighted temperature and the input angular diameter distance. All simulations had an input global baryon fraction of 0.2, so it can be seen that this method seems to slightly underestimate the global baryon fraction. The second cluster had the largest deviation, but this cluster also had significant substructure (see figure 1).

The hydrostatic method [5] can obtain typical accuracy of about 15%, ignoring temperature uncertainties, suggesting that the gas temperature at current levels of sensitivity could be estimated to an accuracy of roughly 20%. Using this temperature rather than the X-ray emission-weighted temperature should allow a determination of the gas fraction of the cluster to an accuracy of 30%, assuming an angular diameter distance. If the true gas fraction is known, this allows a determination of the angular diameter distance to a similar accuracy.

We also did preliminary tests on real data from two high redshift clusters, MS1054 and

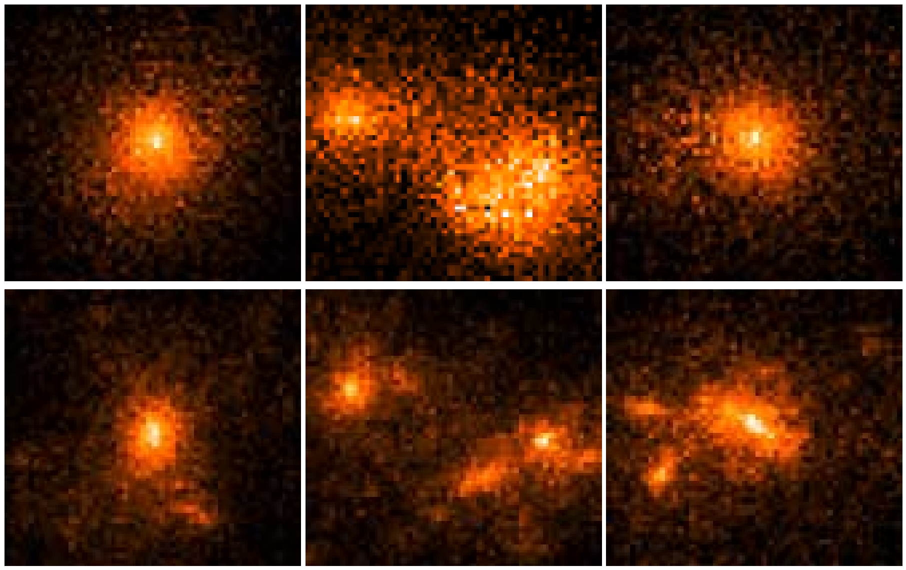

Figure 1: Images of three simulated clusters. Top panels show the SZE maps while lower panels show the projected mass maps. Each map is roughly 10' on a side.

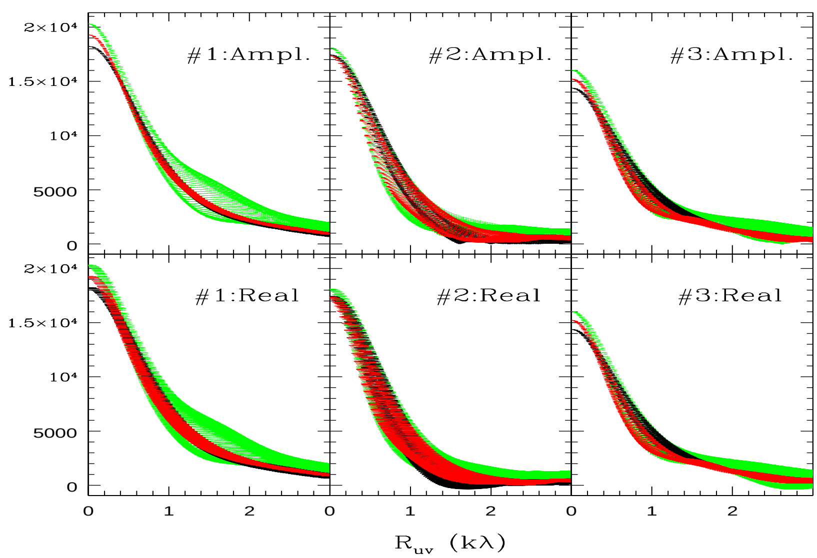

Figure 2: Radial profiles of the Fourier transforms of the SZE (black) gas (red) and total mass (green) for three simulated clusters. Top panels show the Fourier amplitudes and lower panels only the real parts.

CL0016. The weak lensing data were supplied for MS1054 by A. Tyson and D. Wittman at Lucent Technologies and for CL0016 by F. Castander and his collaborators (especially D. Clowe). The SZE data were from observations at OVRO by John Carlstrom, Marshall Joy and collaborators as part of an ongoing SZE imaging project.

For these two clusters, we obtained gas fractions that are consistent with the gas fractions of Grego et al. (2000), with comparable uncertainties. Within the fairly large uncertainties, the derived gas temperatures were also in agreement with X-ray emission-weighted temperature, providing more evidence that these clusters are indeed high-redshift massive clusters.

5 Summary, Future Work

We have outlined a method to compare interferometric SZE measurements and weak lensing observations in a model-independent way. Preliminary work on both real and simulated clusters indicates that this method holds much promise. In principle, it is possible to obtain both direct distances and gas temperatures for distant clusters by this method, providing an independent check on X-ray determined properties of clusters.

Much work remains to be done in testing this method, including extending it beyond the weak lensing limit and exploring the possibilities of loosening the assumption of isothermality. However, it is already clear that, as SZE capabilities improve and weak lensing fields get larger and deeper, the comparison of SZE and weak lensing properties should yield a wealth of information on galaxy clusters.

Acknowledgments

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