Inflation after BICEP2

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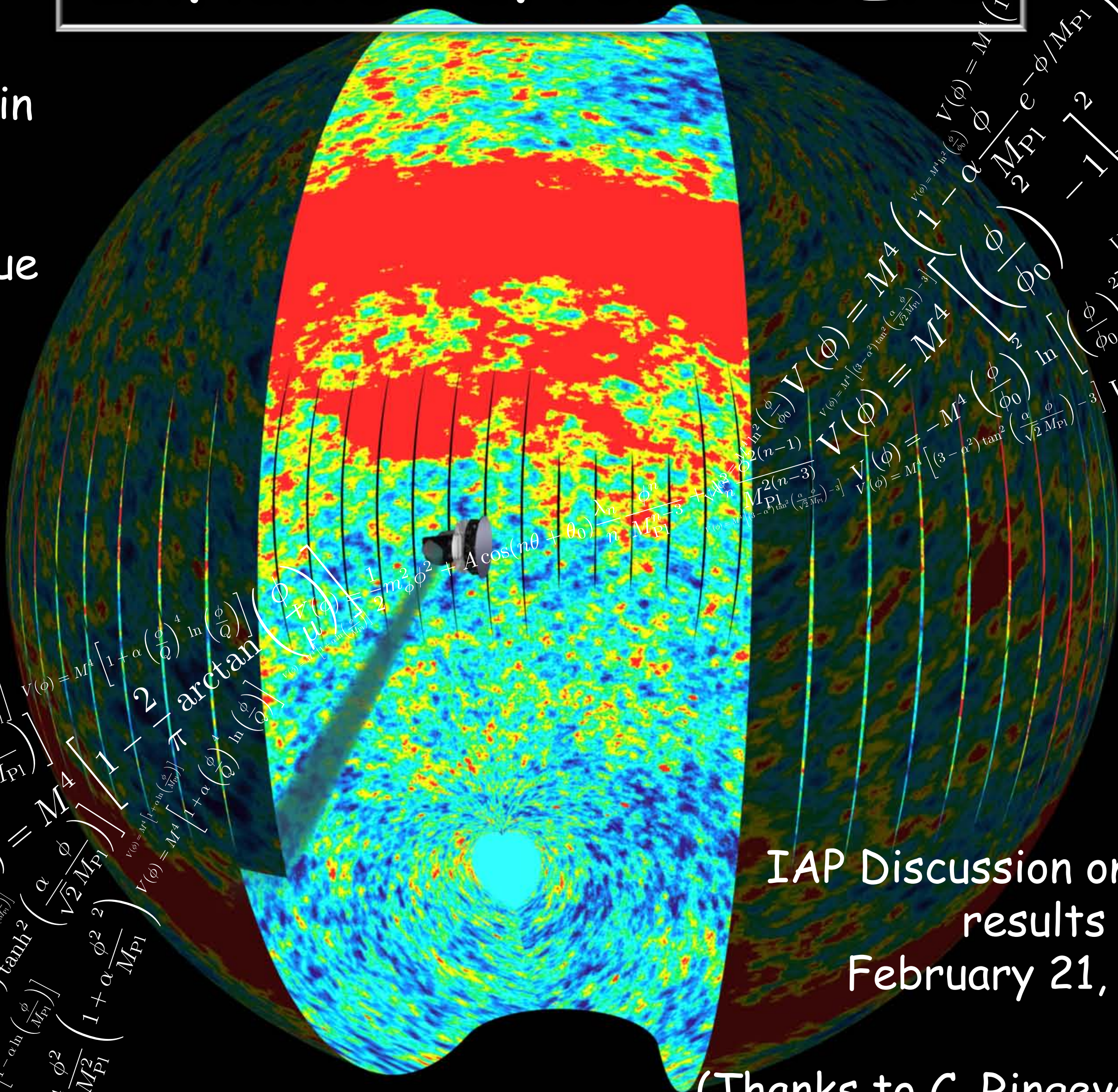


IAP Discussion on BICEP2 results
February 21, 2014

(Thanks to C. Ringeval & V. Vennin for emergency figures, plots etc ...)

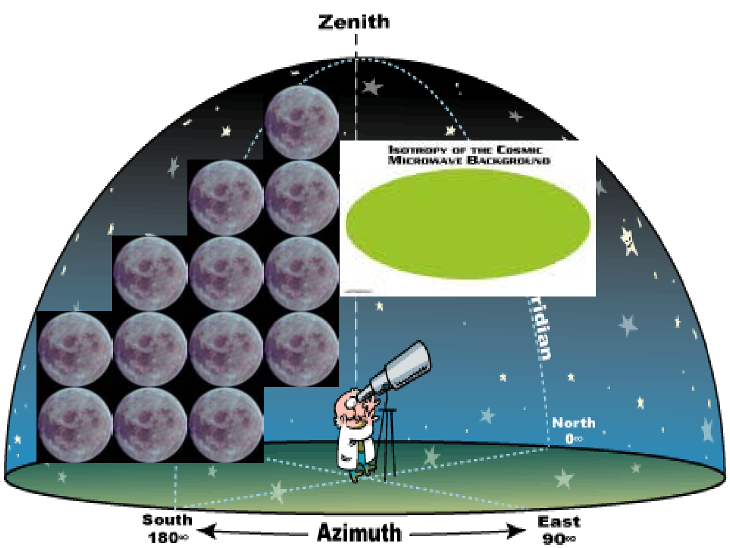
Inflation solves the standard puzzles of the HBB phase

Inflation is a phase of accelerated expansion taking place in the very early Universe. The scale factor is such that

$$\frac{\mathrm{d}^2 a}{\mathrm{d}t^2} > 0$$

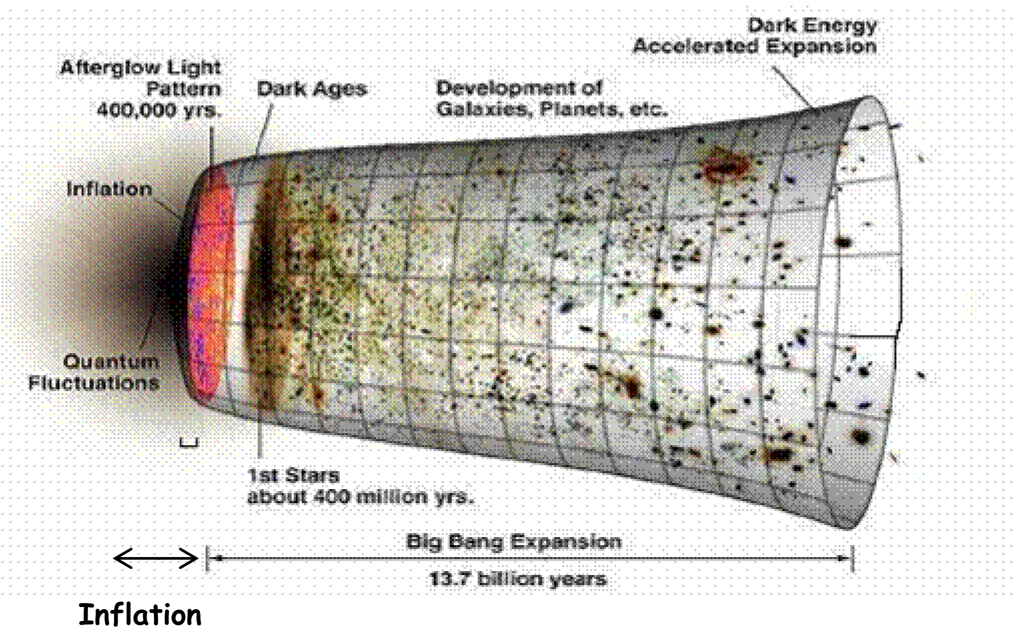
This assumption allows us to solve several problems of the standard hot Big Bang model:

- **Horizon problem**
- **Flatness**
- **Monopoles problem ...**



Inflationary energy scale

- Inflation does not replace the Hot Big Bang model. It is a new ingredient which completes the standard model. It takes place before the Hot Big Bang phase



$$\rho_{\rm now} \simeq 10^{-47}{\rm GeV}^4$$

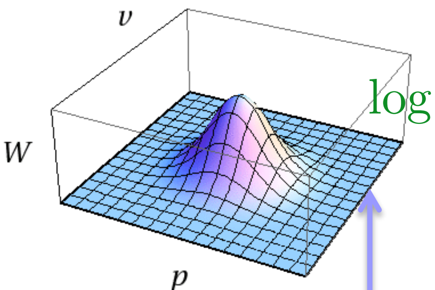
$$\rho_{\rm LHC} \simeq (100{\rm GeV})^4$$

Inflation

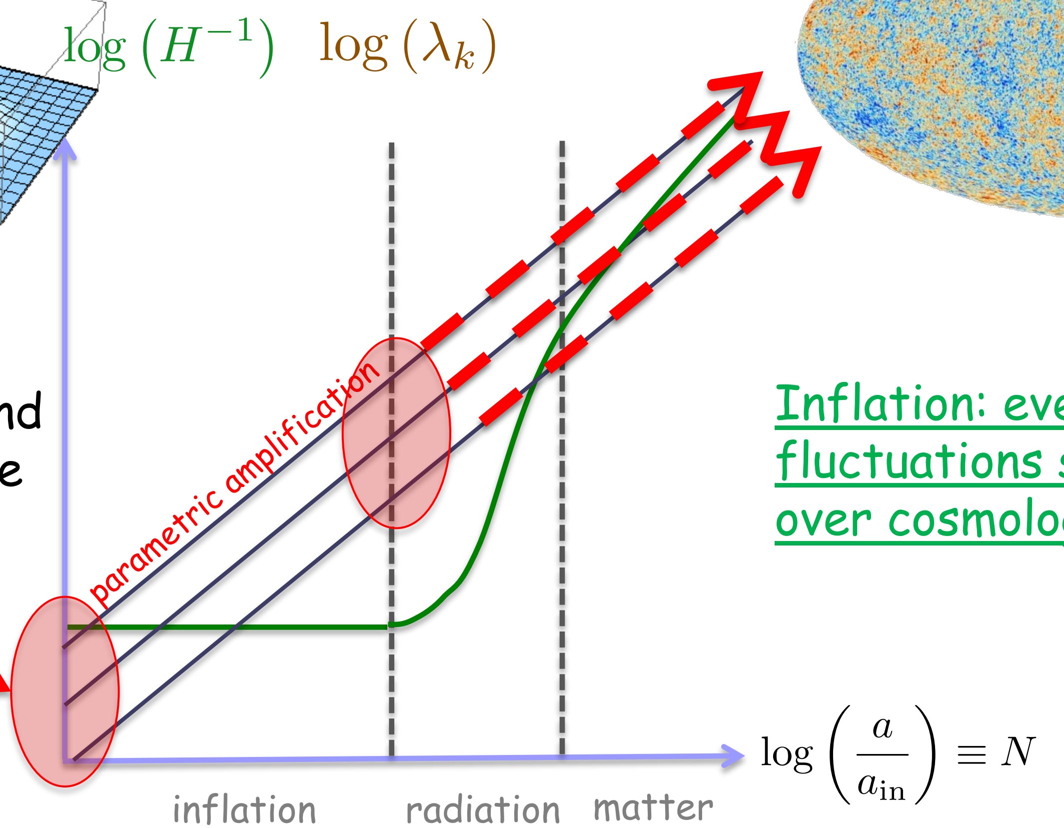
$$\rho_{\rm inf} \simeq (10^{16}{\rm GeV})^4$$

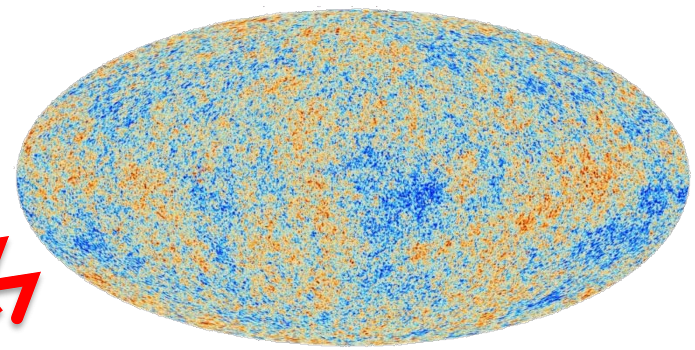
- The energy scale of inflation can a priori be in a wide range of values ~ 12 orders of magnitude! This is probably the most important inflationary quantity!

Quantum fluctuations as seeds of CMB anisotropy and large scale structures

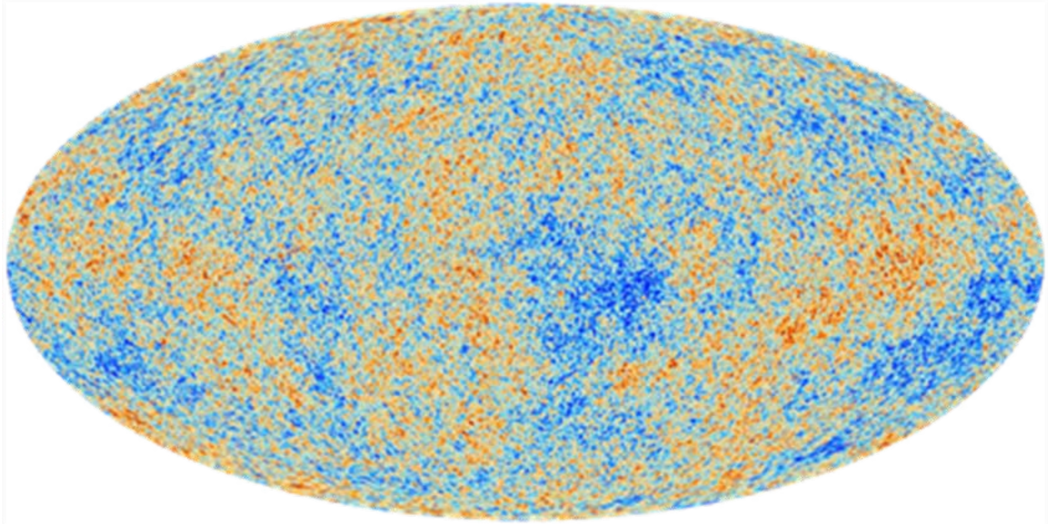


Initial quantum state: ground state of the harmonic oscillator





Inflation: everything is quantum fluctuations stretched over cosmological scales …!



$$\frac{\delta T}{T}(\vec{e}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\vec{e}) \longrightarrow \left\langle \frac{\delta T}{T}(\vec{e}_1) \frac{\delta T}{T}(\vec{e}_2) \right\rangle = \sum_{\ell=2}^{+\infty} \frac{2\ell+1}{4\pi} C_\ell P_\ell(\cos\theta)$$

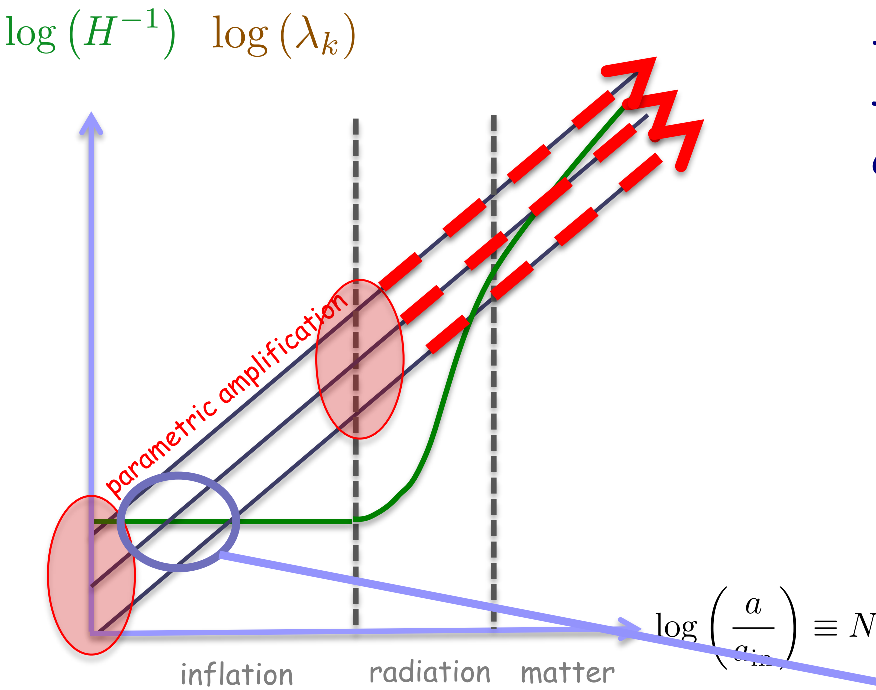
with

$$C_\ell = \langle a_{\ell m} a^*_{\ell m} \rangle = \int_0^{+\infty} \frac{\mathrm{d}k}{k}\, j_\ell^2\left(k r_{\mathrm{lss}}\right) T\left(k; \theta_{\mathrm{stand}}\right) \mathcal{P}_\zeta\left(k; \theta_{\mathrm{reh}}, \theta_{\mathrm{inf}}\right)$$

- $j_\ell^2\left(k r_{\mathrm{lss}}\right)$: Translate 3d into 2d
- $T\left(k; \theta_{\mathrm{stand}}\right)$: Describes the evolution of the pertubations when they re-enter the Hubble radius
- $\mathcal{P}_\zeta\left(k; \theta_{\mathrm{reh}}, \theta_{\mathrm{inf}}\right)$: Inflationary power spectrum

$$k^3 \left|\zeta_{\vec{k}}\right|^2$$

Calculating the two-point correlations: slow-roll parameters



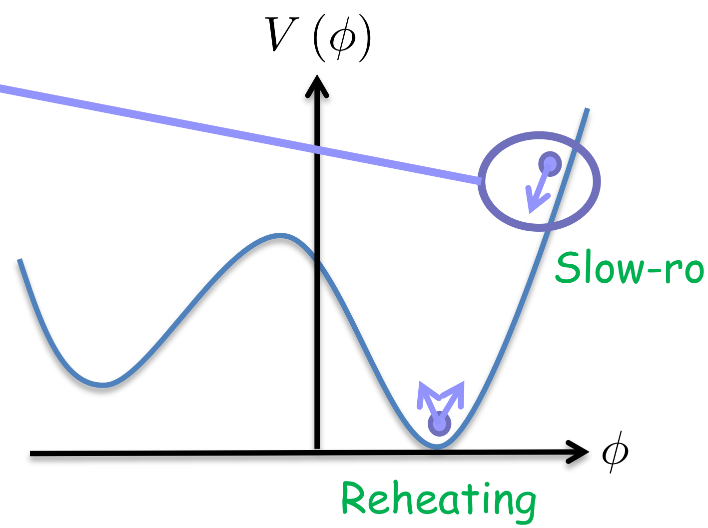
The slow-roll parameters are the "small parameters" of a perturbative calculation of the power spectrum

$$\epsilon_0 \propto H^{-1} \simeq \text{constant}$$

$$\epsilon_{n+1} = \frac{\mathrm{d}\ln|\epsilon_n|}{\mathrm{d}N}, \quad n \geq 0$$

$$\epsilon_1 \simeq \frac{1}{2M_{\rm Pl}^2}\left(\frac{V_\phi}{V}\right)^2$$

$$\epsilon_2 \simeq \frac{2}{M_{\rm Pl}^2}\left[\left(\frac{V_\phi}{V}\right)^2 - \frac{V_{\phi\phi}}{V}\right]$$



$$\mathcal{P}_\zeta = \frac{H^2}{\pi\epsilon_1 m_{\rm Pl}^2}\left[1 - 2\left(C+1\right)\epsilon_1 - C\epsilon_2 - \left(2\epsilon_1+\epsilon_2\right)\ln\left(\frac{k}{k_{\rm P}}\right)\right]$$

$$\mathcal{P}_h = \frac{16H^2}{\pi m_{\rm Pl}^2}\left[1 - 2\left(C+1\right)\epsilon_1 - 2\epsilon_1\ln\left(\frac{k}{k_{\rm P}}\right)\right]$$

- The amplitude is controlled by H
- For the scalar modes, the amplitude also depends on ε_1
- $C \sim -0.7$

The power spectra are scale-invariant plus logarithmic corrections the amplitude of which depend on the sr parameters, ie on the microphysics of inflation

Consistency relation:

$$r = \frac{T}{S} \equiv \frac{\mathcal{P}_h}{\mathcal{P}_\zeta} = 16\epsilon_1 = -8n_{\rm T}$$

Gravitational waves are subdominant

The spectral indices are given by

$$n_{\rm S} - 1 \equiv \frac{{\rm d}\ln\mathcal{P}_\zeta}{{\rm d}\ln k}\,, \quad n_{\rm T} \equiv \frac{{\rm d}\ln\mathcal{P}_h}{{\rm d}\ln k}$$

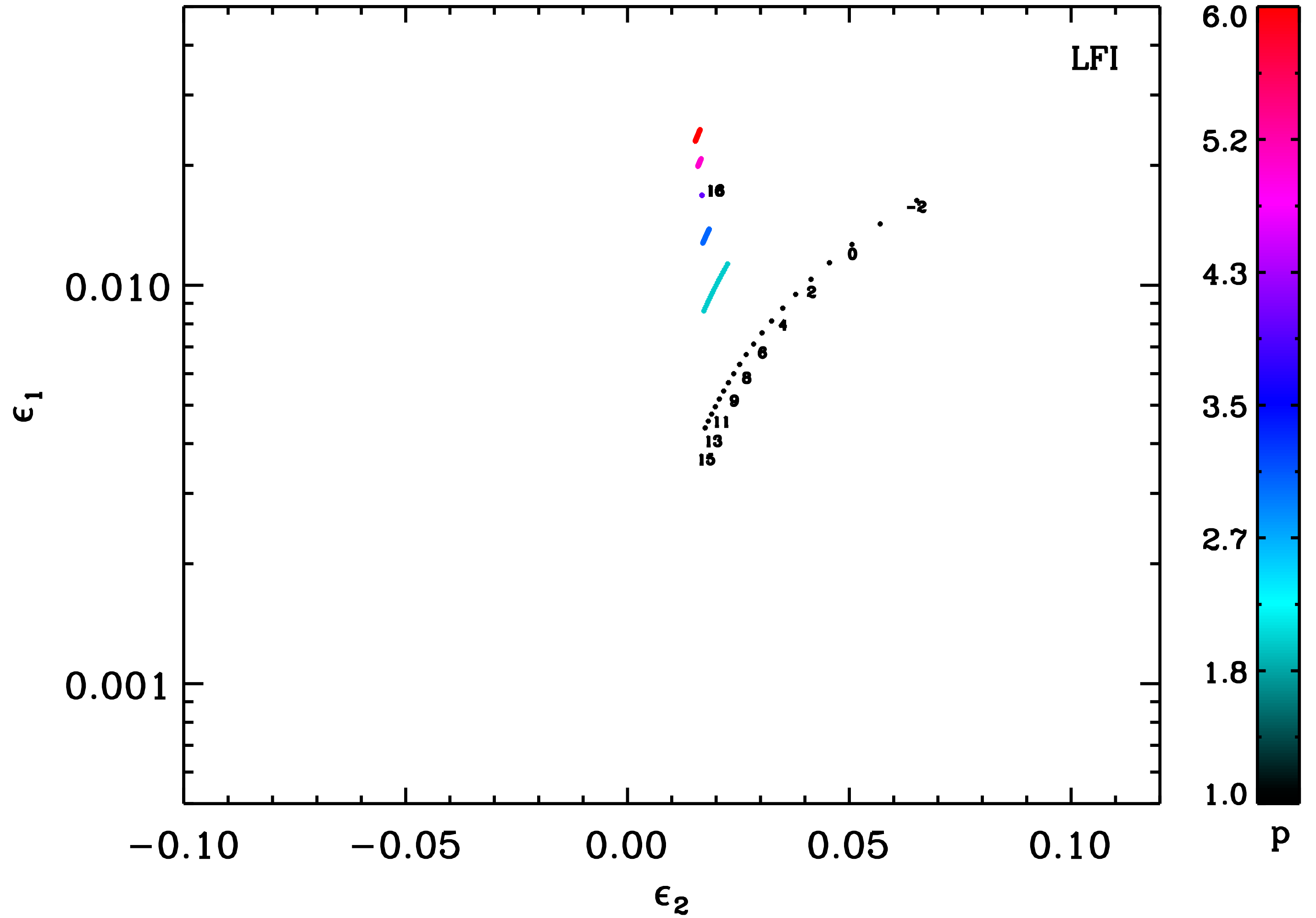
$$n_{\rm S} - 1 = -2\epsilon_1 - \epsilon_2\,, \quad n_{\rm T} = -2\epsilon_1$$

The running, i.e. the scale dependence of the spectral indices, of dp and gw are

$$\alpha_{\rm S} \equiv \frac{{\rm d}^2\ln\mathcal{P}_\zeta}{{\rm d}\left(\ln k\right)^2} \qquad \alpha_{\rm T} \equiv \frac{{\rm d}^2\ln\mathcal{P}_h}{{\rm d}\left(\ln k\right)^2}$$

$$\alpha_{\rm S} = -2\epsilon_1\epsilon_2 - \epsilon_2\epsilon_3$$
$$\alpha_{\rm T} = -2\epsilon_1\epsilon_2$$

An example: « large field inflation » $V(\phi) = M^4 \left(\frac{\phi}{M_{\rm Pl}}\right)^p$



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Planck results in brief:

$$100\,\Omega_{\mathcal{K}} = -0.05^{+0.65}_{-0.66}$$

$$\alpha^{(2,2500)}_{\mathcal{R}CDI} \in [-0.093, 0.014]$$

$$n_{\mathrm{S}} = 0.9603 \pm 0.0073$$

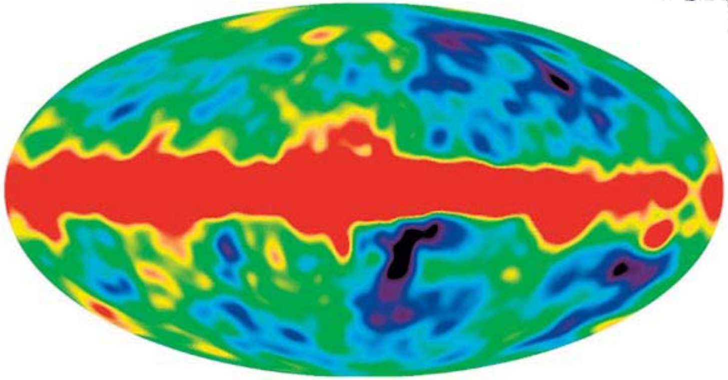
$$\frac{\mathrm{d}n_{\mathrm{S}}}{\mathrm{d}\ln k} = -0.0134 \pm 0.009$$

$$f^{\mathrm{loc}}_{\mathrm{NL}} = 2.7 \pm 5.8$$

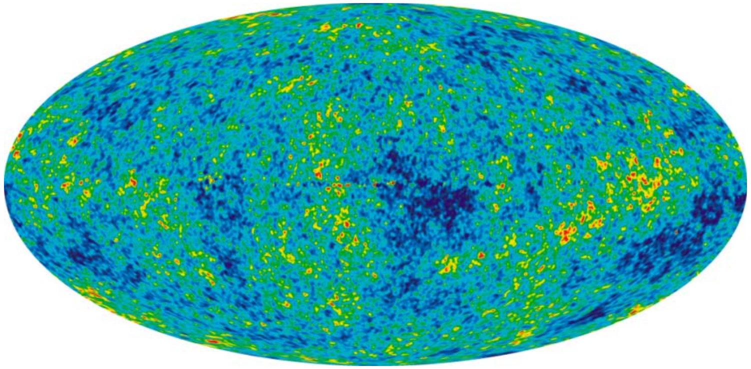
$$f^{\mathrm{eq}}_{\mathrm{NL}} = -42 \pm 75$$

$$f^{\mathrm{ortho}}_{\mathrm{NL}} = -25 \pm 39$$

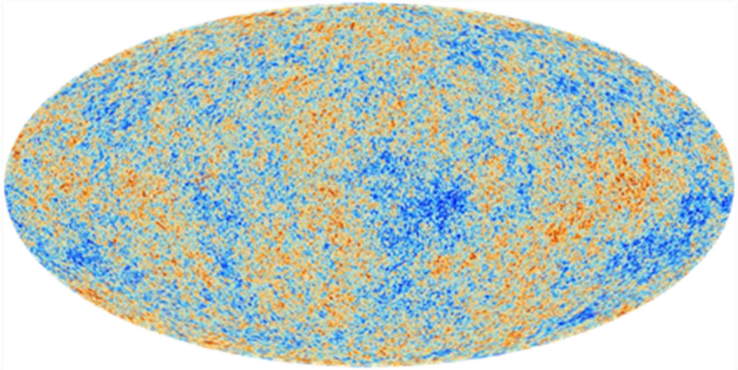
Flat universe with adiabatic, Gaussian and almost scale invariant fluctuations



COBE (1992)



WMAP (2003)



Planck (2013)

Message 1: the energy scale of inflation

Before BICEP2

$$\mathcal{P}_h \simeq \left(\frac{H}{m_{\rm Pl}}\right)^2 < \mathcal{O}(1)\left(\frac{\delta T}{T}\right)^2 \simeq 10^{-10} \rightarrow$$

Upper bound on the energy scale of inflation ~ less than the GUT scale

Message 1: the energy scale of inflation

Before BICEP2

$$\mathcal{P}_h \simeq \left(\frac{H}{m_{\rm Pl}}\right)^2 < \mathcal{O}(1)\left(\frac{\delta T}{T}\right)^2 \simeq 10^{-10}$$

→ Upper bound on the energy scale of inflation ~ less than the GUT scale

After BICEP2

$$\mathcal{P}_h \simeq \left(\frac{H}{m_{\rm Pl}}\right)^2 \simeq 0.2\left(\frac{\delta T}{T}\right)^2 \simeq 0.2\times 10^{-10}$$

→ Energy scale of inflation measured to be ~ the GUT scale

$$H \simeq 1.23\left(\frac{r}{0.2}\right)^{1/2} 10^{14}\,\mathrm{GeV}$$

$$\rho^{1/4} \simeq 2.26\left(\frac{r}{0.2}\right)^{1/4} 10^{16}\,\mathrm{GeV}$$

Message 2: first derivative of the potential

Before BICEP2

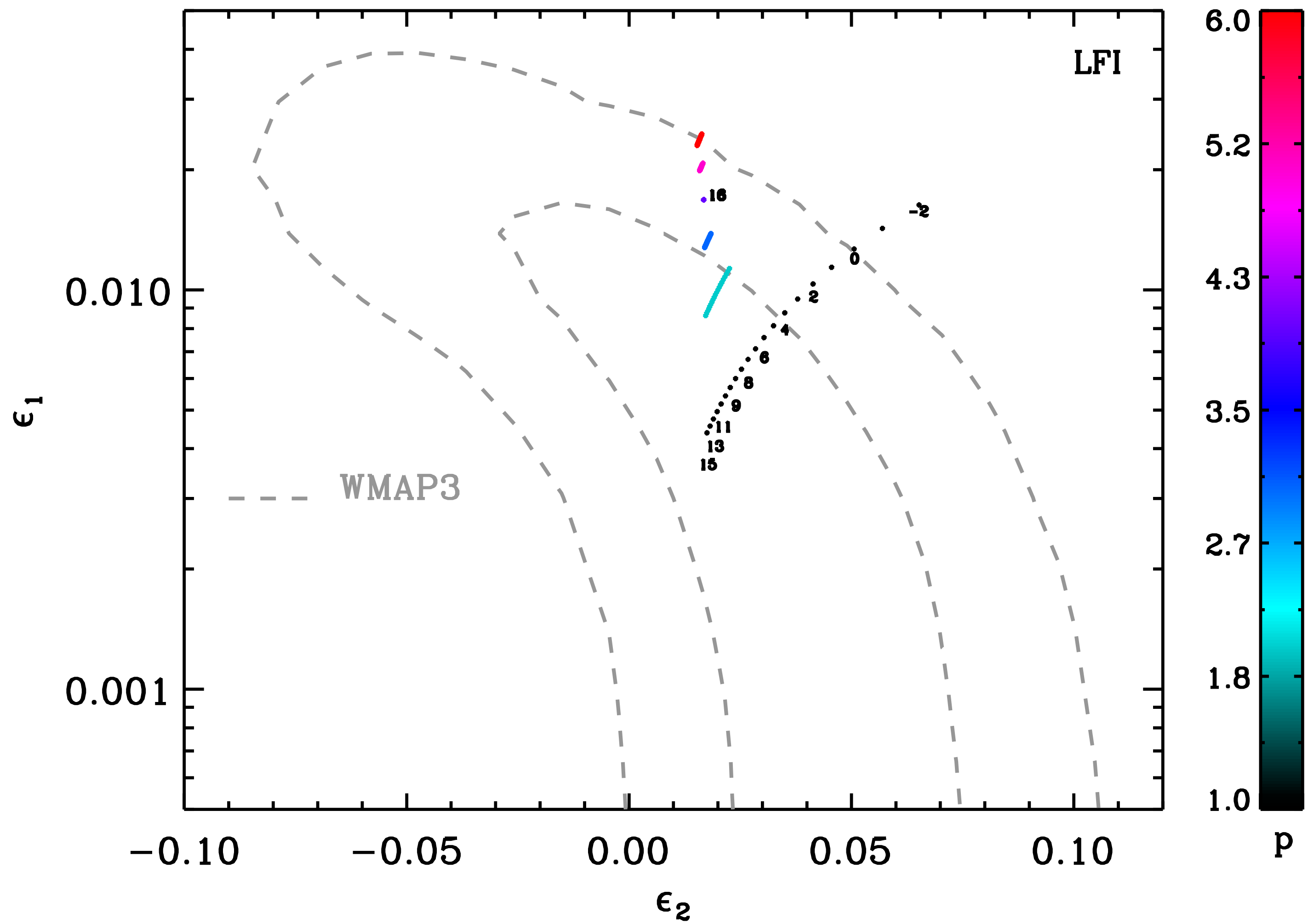
$$r=\frac{T}{S}=16\epsilon_1=\frac{8}{M_{\rm Pl}^2}\left(\frac{V_\phi}{V}\right)^2<\mathcal{O}(1)$$

→ Upper bound on the value of the first derivative

$$n_{\rm S}-1=-2\epsilon_1-\epsilon_2\simeq 0.96$$

→ Second sr parameter measured!

An example: « large field inflation » $V(\phi) = M^4 \left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^p$



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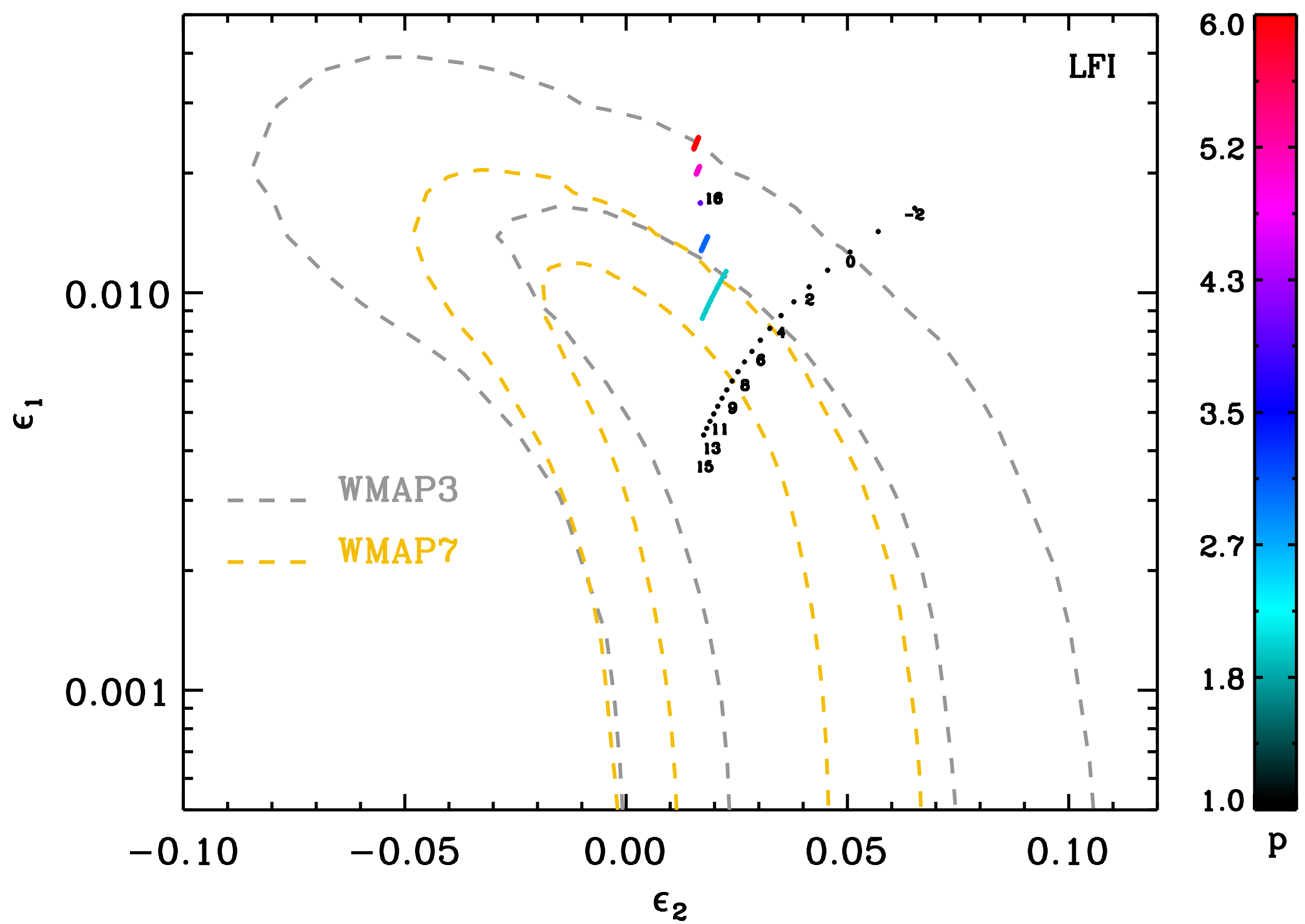

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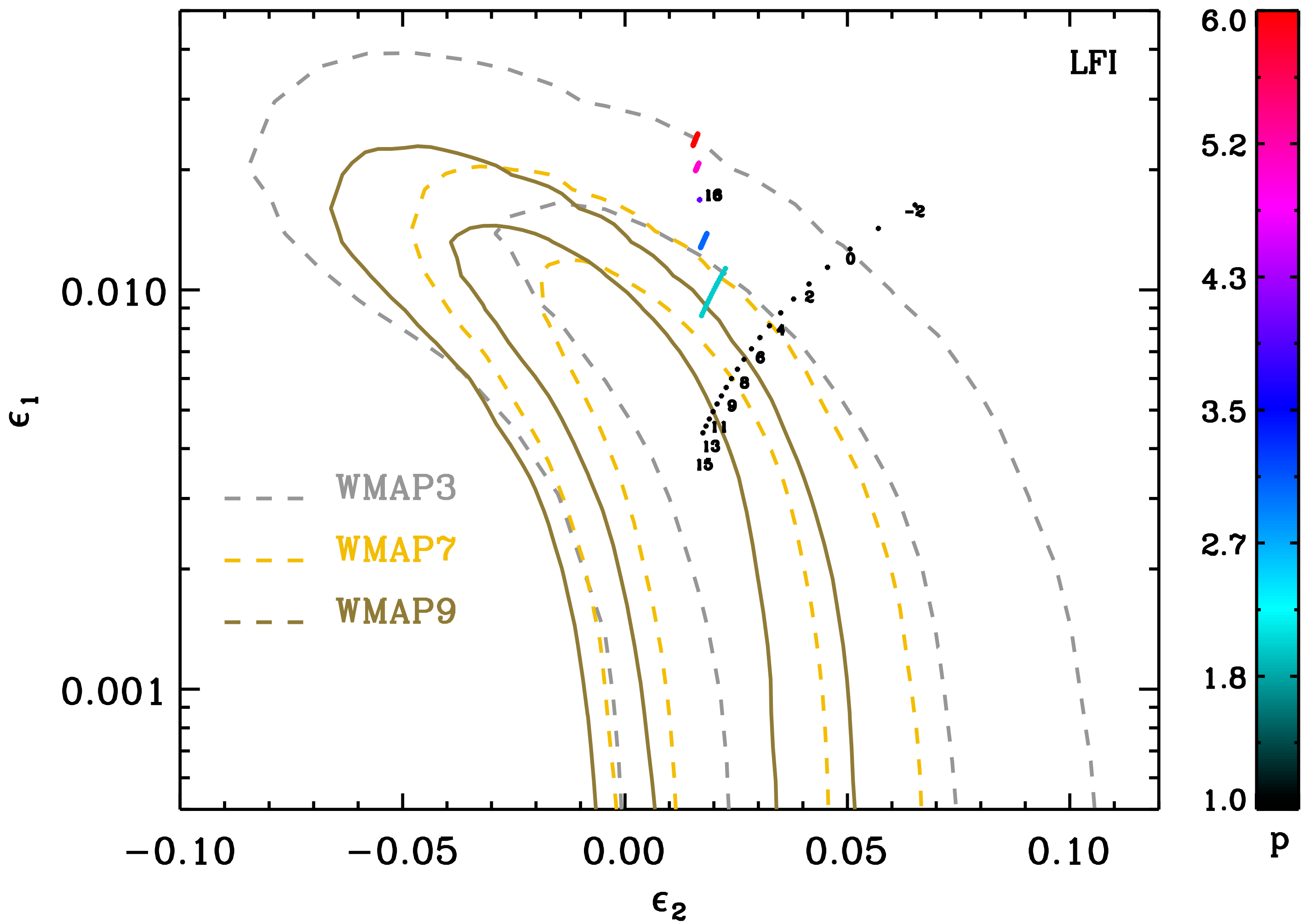

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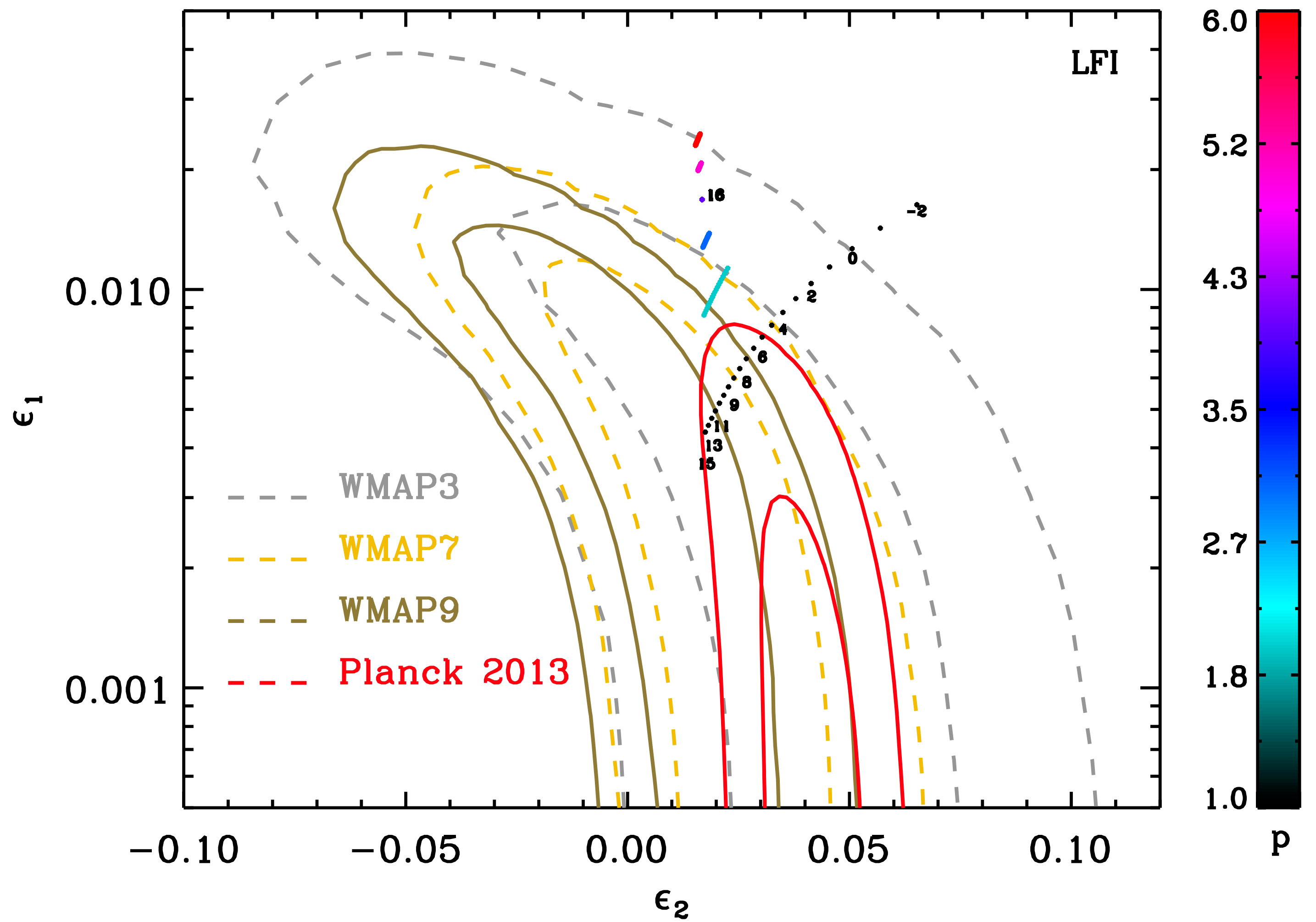

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Message 2: first derivative of the potential

Before BICEP2

$$r = \frac{T}{S} = 16\epsilon_1 = \frac{8}{M_{\rm Pl}^2}\left(\frac{V_\phi}{V}\right)^2 < \mathcal{O}(1)$$

→ Upper bound on the value of the first derivative

$$n_{\rm S} - 1 = -2\epsilon_1 - \epsilon_2 \simeq 0.96$$

→ Second derivative measured!

After BICEP2

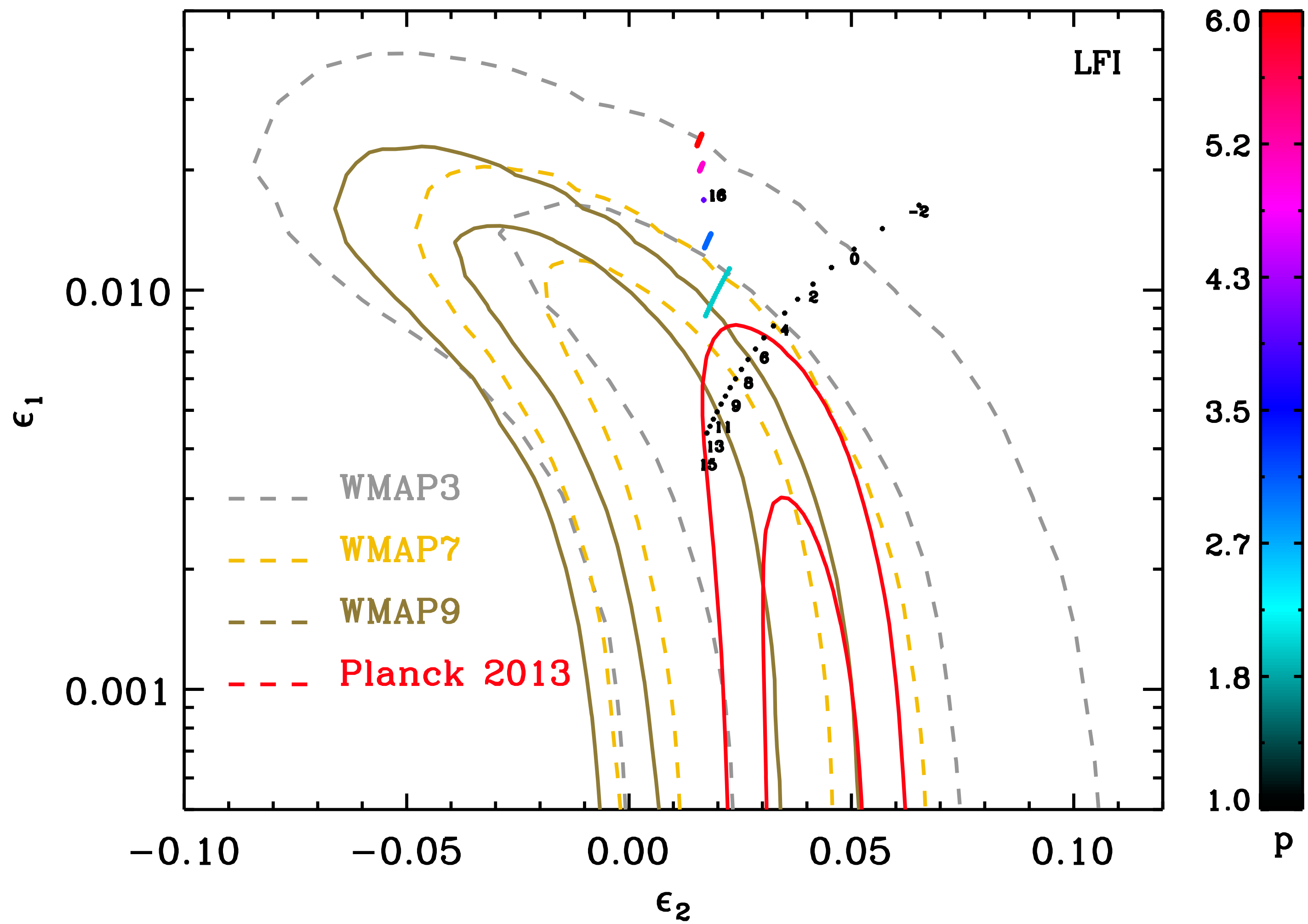
$$r = \frac{T}{S} = 16\epsilon_1 = \frac{8}{M_{\rm Pl}^2}\left(\frac{V_\phi}{V}\right)^2 = 0.2$$

→ First derivative measured!

$$n_{\rm S} - 1 = -2\epsilon_1 - \epsilon_2 \simeq 0.96$$

→ Second derivative measured but different value

An example: « large field inflation » $V(\phi) = M^4 \left(\frac{\phi}{M_{\rm Pl}} \right)^p$



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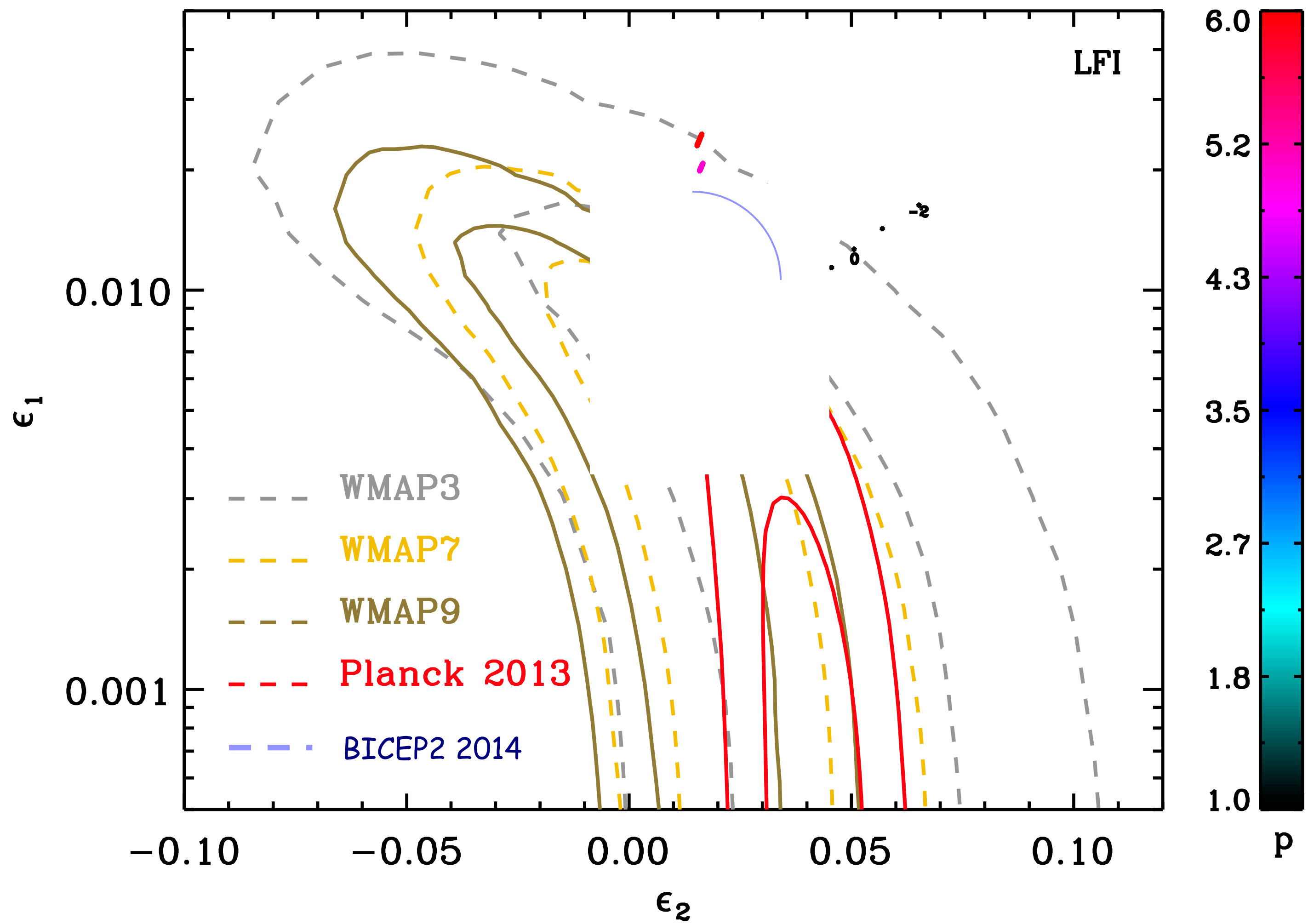
An example: « large field inflation » $$V(\phi) = M^4 \left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^p$$



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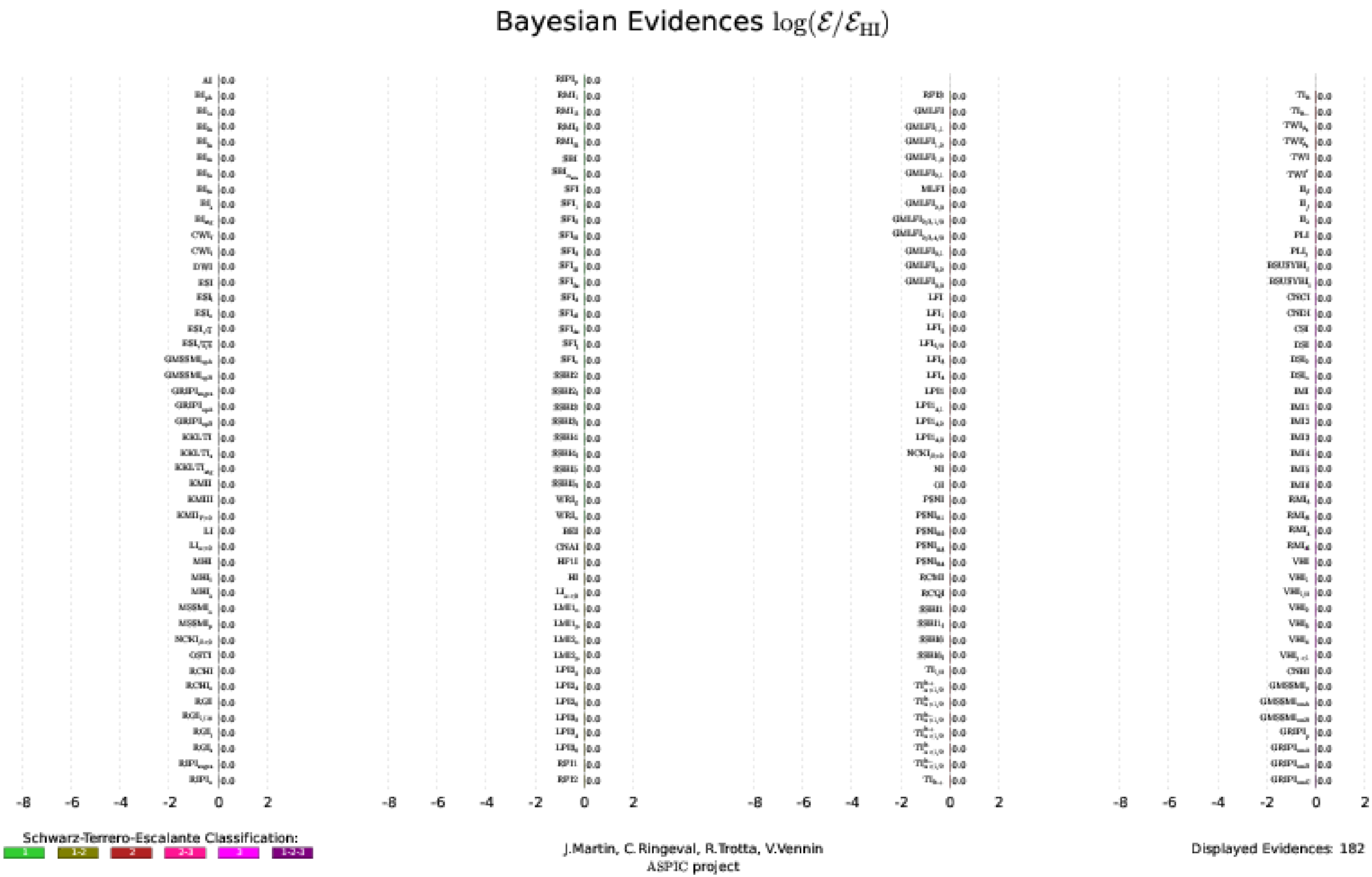
Message 3: shape of the potential

Before BICEP2



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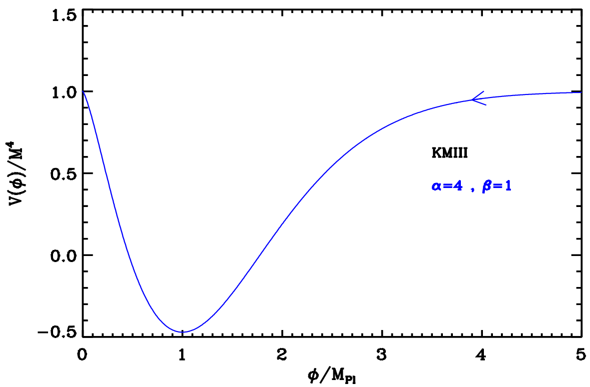
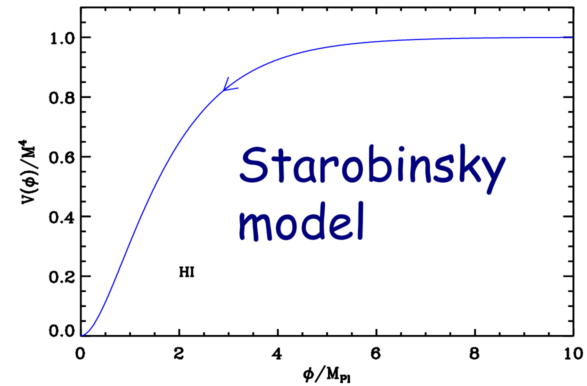
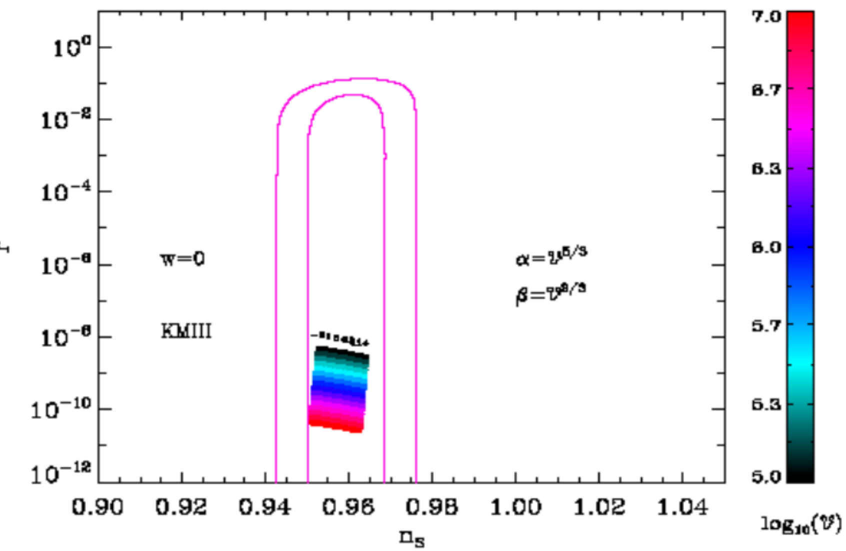
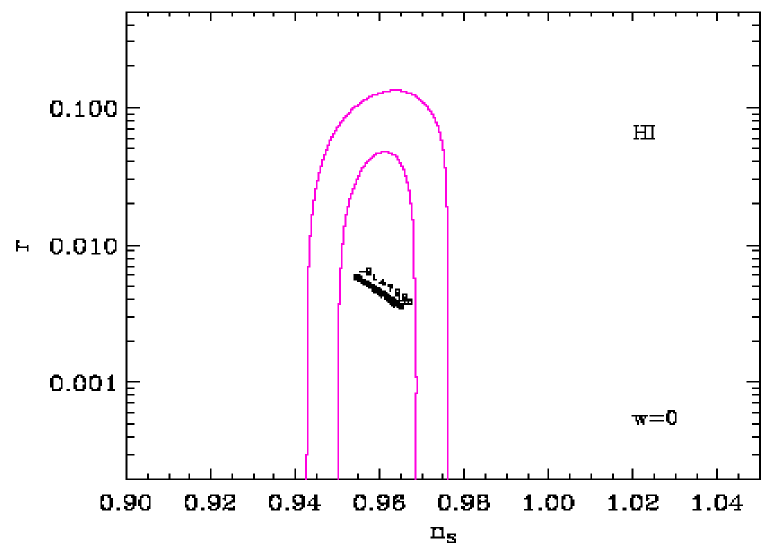


Plateau inflation



Starobinsky model





Message 3: shape of the potential

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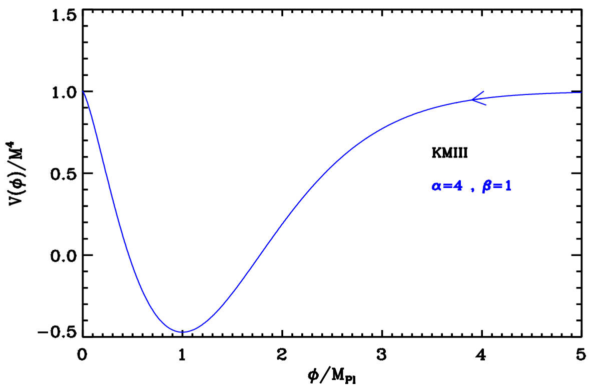

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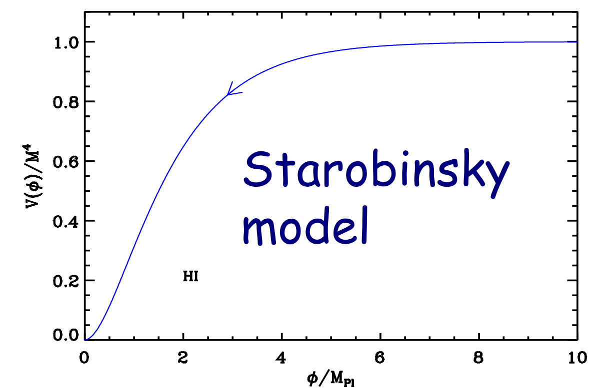
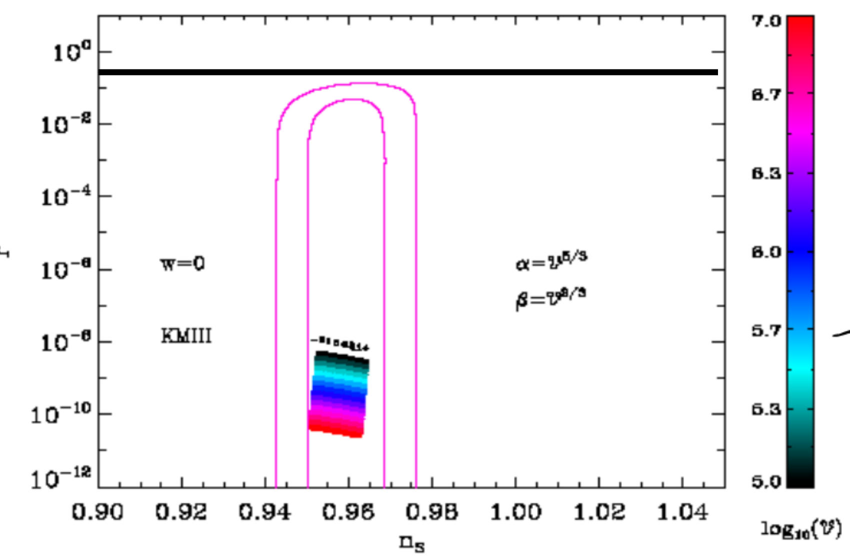
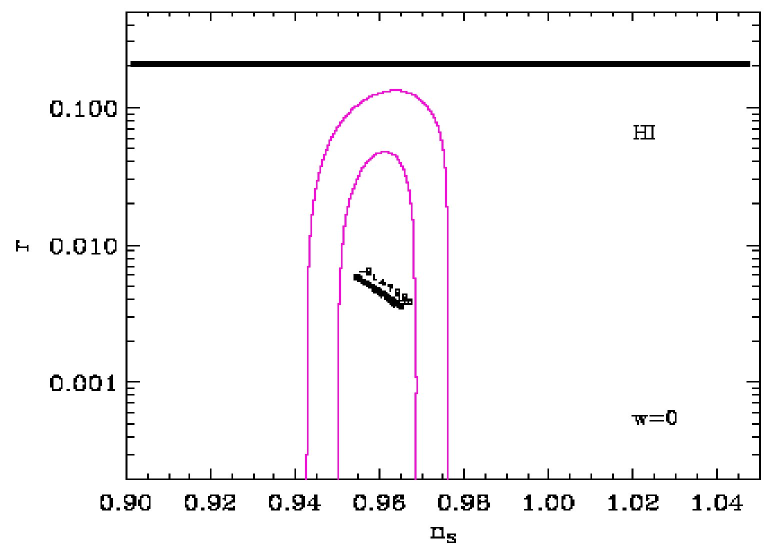


After BICEP2





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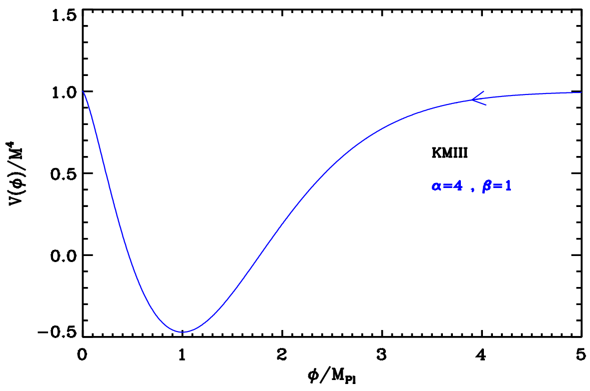

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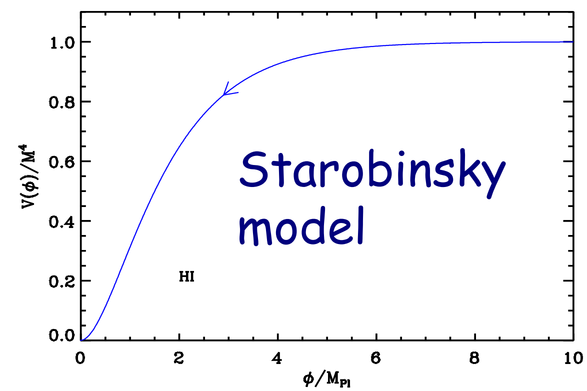
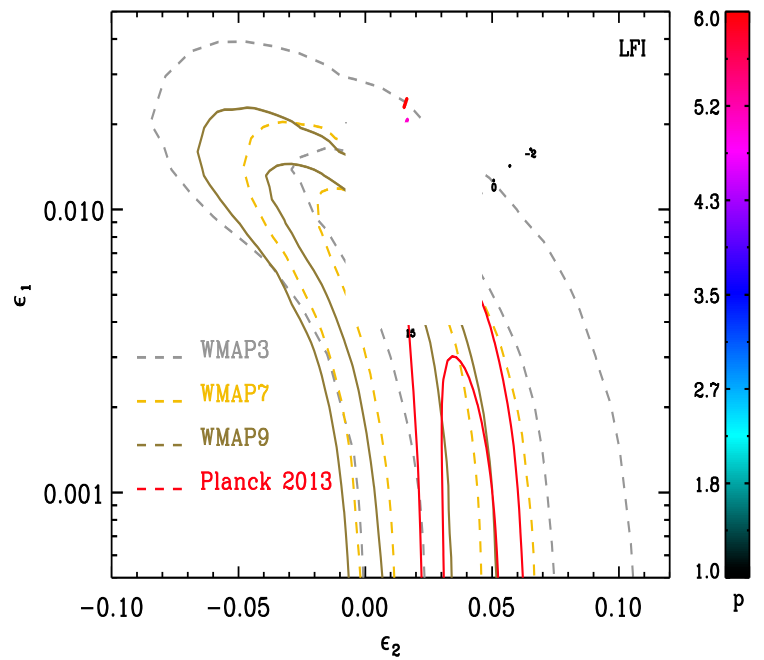


Starobinsky model

After BICEP2

$$V(\phi) = \frac{1}{2} m^2 \phi^2$$



Message 4: more complicated class of models

Before BICEP2

Simplest models favored (ie more complicated not needed) because no isocurvature modes, no NG etc …

Message 4: more complicated class of models?

Before BICEP2

Simplest models favored (ie more complicated not needed) because no isocurvature modes, no NG etc …

After BICEP2

- K-inflation $r = -8 n_{\mathrm{T}} c_{\mathrm{S}} \, , \quad c_{\mathrm{S}} < 1$
- Multiple field inflation $r = -8 n_{\mathrm{T}} c_{\mathrm{S}} \sin^2 \Theta$

Still true!

Message 5: model building issues

Before BICEP2

No problem in principle, lot of activities trying to relate the Higgs with the inflaton

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After BICEP2

- New physics at the GUT scale, coupling with the Higgs??
- Difficult because of the Lyth bound: $\Delta\phi \simeq 3.2 \left(\frac{r}{0.1}\right)^{1/2} M_{\rm Pl}$
- Break-down of EFT??

Criticisms on inflation after Planck & BICEP2

- Inflation uses a scalar field but nobody has ever seen a scalar field

  - Higgs at the LHC

- No prediction of inflation, only post-dictions …

  - ns ~ 0.96 ≠1

  - Presence of primordial gravity waves

- Alternatives to inflation??

  - Ekpyrotic model predicts no detectable gw …

  - String gas cosmology, bouncing models??

Have we finally proven inflation????

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NO!

Have we finally proven inflation????

NO!

We need to check the consistency relations

$$n_{\mathrm{T}} = -\frac{r}{8} \simeq -0.025$$

$$\alpha_{\mathrm{T}} = \frac{r}{8}\left[\frac{r}{8} + (n_{\mathrm{S}} - 1)\right] \simeq 0.0016$$

We need to measure the tensor spectral index; since r is large, this seems feasible

- First calculation of gw production in curved spacetime, L. P. Grishchuk (1975)
- First application to an inflationary background, A. Starobinsky (1979)